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Conjugate-image operation of transistor amplifiers

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CONJUGATE-IMAGE OPERATION OF TRANSISTOR AMPLIFIERS

by

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I. INTRODUCTION

A. General Discussion

The discovery of the transistor was announced in 1948 by Bardeen and Brattain (3). The original transistor was called a point-contact transistor and consisted of a small wafer of n-type germanium with two whiskers pressed against the surface. With leads attached to each of the whiskers and a lead attached to the wafer the transistor is thus a three terminal device. The terminals are designated the collector, the emitter and the base.

The relatively few years since 1948 have seen a great deal of work in the general area of transistor electronics. In addition to germanium, silicon is used as a raw material for making transistors. Transistors now come in a wide variety of types such as the junction transistor, the transistor tetrode and the intrinsic-barrier transistor.

The type of transistor which seems to be the most widely used is the junction transistor. It is capable of relatively high power gain, is rugged, is more stable than the point-contact transistor, is small in size and has low power requirements. The theory of junction transistors is fairly well understood and they can be built to certain prescribed electrical characteristics. Junction transistors come in two basic types, either the pnp transistor or the npn transistor, also the semiconductor can be either silicon or germanium. A pnp junction transistor consists of two regions of p-type semiconductor separated by a thin region of n-type semiconductor. The npn transistor just reverses the arrangement of the semiconductor types.

In terms of signal the transistor amplifier is operated as a two terminal pair device with one terminal being common to both the input and output. Thus there are six possible methods of operation, only three of which are practical: the common-base with emitter input and collector output, the common-emitter with base input and collector output, and the common-collector with base input and emitter output. The transistor is a general two terminal pair network; that is, it has finite input and output impedances and provides transmission in both forward and reverse directions.

A junction transistor when operated common-emitter or common-base has a low input impedance and a high output impedance. These impedances are pure resistances at direct current, but they can become complex at fairly low frequencies. The input impedance is determined by transistor small signal parameters and the externally connected load. The output impedance depends upon the source impedance and the transistor small signal parameters.

Maximum power will be transferred into the transistor when the source impedance is the conjugate of the input impedance and maximum power will be delivered to the load impedance when the load impedance is the conjugate of the output impedance. This is called conjugate-image matching or conjugate-image operation. Operation in this manner maximizes the available power gain of a transistor amplifier. Available power gain of an amplifier is defined as the ratio of the power output to the maximum available power input.

Conjugate-image operation is not an easy thing to accomplish for complex impedances. The conjugate of a physically realizable impedance function, that varies with frequency, can not be approximated over a continuous range of frequencies by a physically realizable impedance function. Thus conjugate-image operation can occur only at a single frequency, however conjugate-image matching can be approximated over a band of frequencies with a reflection factor greater than zero but less than some maximum. The maximum depends on the impedance to be matched.

This thesis pertains to the conjugate-image operation of junction transistor amplifiers.

B. Review of Literature

Since the discovery of the transistor in 1948 a large number of articles and several books have been devoted to transistors and transistor circuits. Several of the earlier articles discussing amplifiers consider the transistor as a resistive device, that is operating at frequencies where reactive effects can be neglected. Later papers, such as that of Bruun (6), treat the more general case, where reactive effects are considered.

The equivalent circuits proposed for the junction transistor have become increasingly complex as the desired accuracy and frequency range have increased. These equivalent circuits are usually based on the solution of a one dimensional diffusion equation plus some extra terms not explained by the diffusion process. The concentration of charge carriers in the thin base region of a junction transistor can be determined from the solution of a one dimensional diffusion equation. The diffusion part of the equivalent circuit or the intrinsic transistor portion is usually approximated by a RC transmission line or a two terminal pair approximation to a RC transmission line and an amplifying device. Chu (9), Zawels (41), Lo et al. (25) and Statz et al. (36) are some of the authors that base their proposed equivalent circuits on the diffusion process.

Roberts (32) in a 1946 article discusses the conjugate-image operation of a network. Equations are developed for determining the source and load impedances that are conjugate to the input impedance and output impedance respectively of the network. These equations are developed in terms of the z-parameters of a general two terminal pair network. Lo et al. (25) has a similar development in his book, although not as detailed as Roberts' treatment.

Bode (4) considers the problem of matching a resistance shunted by parasitic capacitance to a pure resistance. The reflection factor is related to the capacitance and resistance by an integral relationship which indicates that for a given bandwidth the reflection factor can not be made arbitrarily small for a finite parasitic capacitance. Fano (13) considers the case of matching a resistance to an arbitrary load impedance with a lossless network. Bode's problem then is a special case of the more general problem considered by Fano. Matthaei (26) has condensed the main points of Fano's two part paper into a single article of reasonable length.

Carlin and La Rosa (7) (23) approach the problem of broadband matching from a different point of view. They use a lossy matching network to give a reflectionless termination. The power loss then is caused by losses in the matching network and not by reflections. The authors do not say that a lossy reflectionless matching network will be better than a lossless network. However, there are two advantages: in some cases a reflectionless match is necessary; generally there is an economy in the number of elements needed. The difference between lossless matching and reflectionless matching will not be more than three decibels.

For the conjugate-image operation of junction transistors it will generally be necessary to eliminate the feedback from output to input that exists in the transistor itself. This can be done by externally feeding back a signal of equal magnitude and opposite phase to that fed back internally. Gade (15) considers the general problem of feedback in junction transistors. In 1955 three papers appeared on the neutralization of transistor amplifiers, while possessing some differences the papers by Cheng (8), Chu (10) and Stern et al. (38) discuss the same general problem.

Wallace and pietenpol (40) and Fougere (14) treat the maximum power gain of transistor amplifiers when reactive effects can be neglected. Image operation and conjugate-image operation give the same results for this case. Linvill and Schimpf (24), Stern (37) and Pritchard (30)(31) consider the more general case. Linvill and Schimpf (24) treat the stability and the power gain of tetrode transistor amplifiers. Stern's paper (37) is concerned with the stability and power gain of transistors with tuned loads. Pritchard (30)(31) treats the power gain of junction transistors conjugate matched at the output and driven from a resistive source.

Drouilhet (12) considers the prediction of transistor amplifier performance based on the maximum frequency of oscillation of the transistor. The neutralized matched power gain of a transistor should rise at the rate of about six decibels per octave as frequency is decreased from the maximum frequency of oscillation. As frequency is further decreased, the power gain will level off at a value equal to the low frequency matched power gain.

II. CONJUGATE-IMAGE OPERATION

A. Conjugate-Image Immittances

Bode (4) introduces the term immittance and uses it to refer to a function that could be either an impedance or an admittance. The solutions for the source and load terminations for conjugate-image operation are expressed as a source impedance and load admittance. Therefore, when the term conjugate-image immittances is used it will mean the source impedance for conjugate-image operation and load admittance for conjugate-image operation.

The two equations needed to relate input current, input voltage, output current and output voltage for a two-terminal pair network can be expressed in six ways (18) three of which are in general use in transistor circuit literature. They are

$$V_1 = I_1 z_{11} + I_2 z_{12} \qquad 1$$

$$V_2 = I_1 z_{21} + I_2 z_{22} \qquad 2$$

$$I_1 = V_1 y_{11} + V_2 y_{12} \qquad 3$$

$$I_2 = V_1 y_{21} + V_2 y_{22} \qquad 4$$

and

$$V_1 = I_1 h_{11} + V_2 h_{12} \qquad 5$$

$$I_2 = I_1 h_{21} + V_2 h_{22}. \qquad 6$$

The z-, y- and h-parameters are related and expressions can be derived for converting from one set of equations to another. The h-parameters are used in this thesis, since the low input impedance of a junction transistor makes a voltage source equivalent circuit desirable for the input circuit part and the high output impedance makes a current source desirable for the output circuit part. Figure 1 shows the equivalent circuit for a two terminal pair network using h-parameters.

Z_{c1} and Y_{c2} are used to designate the source impedance and load admittance for conjugate-image operation. The asterisk will be used to denote a conjugate quantity, for example Z_{c1}^* is the conjugate of Z_{c1}. Circuit connections defining Z_{c1} and Y_{c2} are shown in Figures 2 (A) and 2 (B). The network in Figure 3 is connected for conjugate-image operation.

If the network N in Figure 2 (A) is replaced by its h-parameter equivalent circuit, the following equations can be written

$$V_1 = I_1 h_{11} + V_2 h_{12} \qquad 7$$

$$0 = I_1 h_{21} + V_2 (h_{22} + Y_{c2}) \qquad 8$$

$$V_1 = I_1 Z_{c1}^* \qquad 9$$

and for Figure 2 (B)

$$0 = I_1 (h_{11} + Z_{c1}) + V_2 h_{12} \qquad 10$$

$$I_2 = I_1 h_{21} + V_2 h_{22} \qquad 11$$

$$I_2 = V_2 Y_{c2}^* . \qquad 12$$

Using the above equations it can be shown that

$$Z_{c1} = r_{11}(\emptyset_r + j\emptyset_x) - jx_{11} \qquad 13$$

and

$$Y_{c2} = g_{22}(\emptyset_g + j\emptyset_b) - jb_{22} \qquad 14$$

where

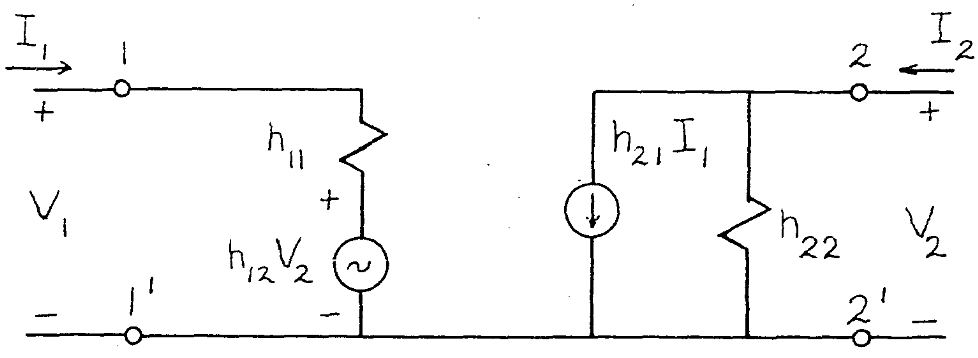
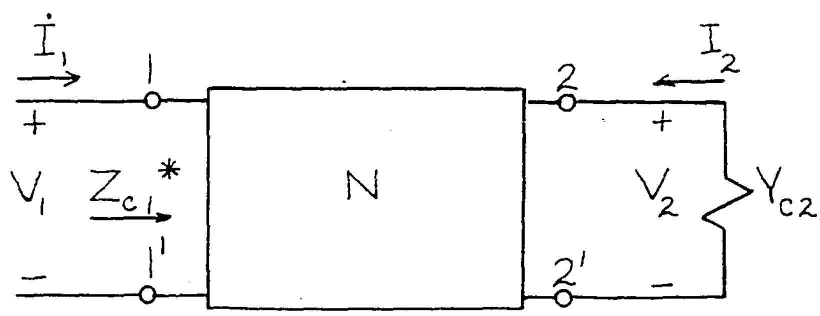
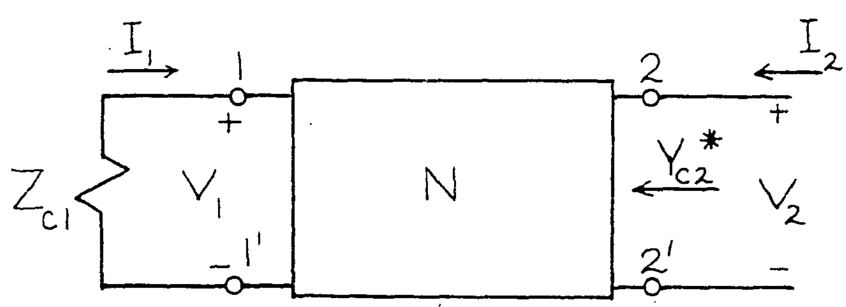


Figure 1. h-parameter equivalent circuit for a two terminal pair network



(A)



(B)

Figure 2. Circuit connections defining Z_{c1} and Y_{c2}

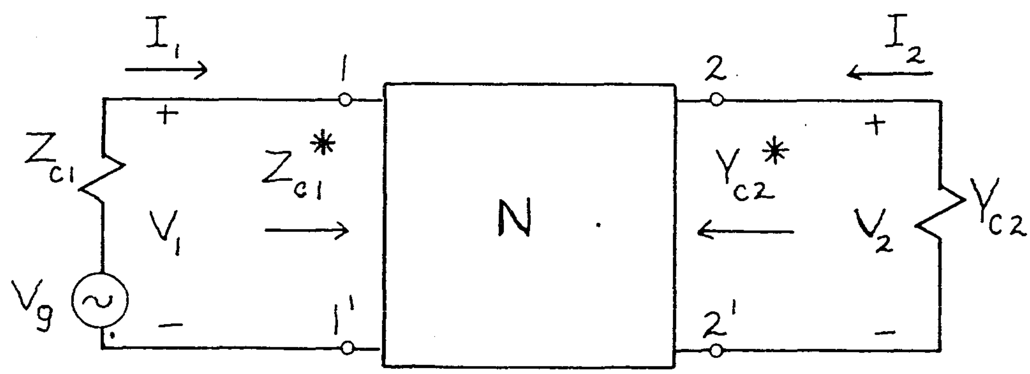


Figure 3. A two terminal pair network connected for conjugate-image operation

$$\phi_x = \phi_b = \frac{p_c q_c}{r_{11} g_{22}} \qquad 15$$

$$\phi_r = \phi_g = \sqrt{1 - \phi_x^2 - \frac{p_c^2 - q_c^2}{r_{11} g_{22}}} \qquad 16$$

$$h_{11} = r_{11} + jx_{11} \qquad 17$$

$$h_{22} = g_{22} + jb_{22} \qquad 18$$

and

$$\sqrt{h_{12} h_{21}} = p_c + jq_c . \qquad 19$$

By proper feedback a two terminal pair network may be neutralized making $h_{12} = 0$. With $h_{12} = 0$, then $p_c = 0$, $q_c = 0$, $\phi_x = \phi_b = 0$ and $\phi_r = \phi_g = 1$. For a neutralized network

$$Z_{c1} = r_{11} - jx_{11} = h_{11}^{*} \qquad 20$$

and

$$Y_{c2} = g_{22} - jb_{22} = h_{22}^{*} . \qquad 21$$

The conjugate-image immittances or the source impedance and load admittance for conjugate-image operation have been expressed in terms of the h-parameters. However, the conjugate-image immittances are independent of the particular small signal parameters used to represent the network. If this were not true, the source impedance and load admittance for maximum available power gain could be changed by changing the network representation. Changing the network representation does not involve changing the network, and therefore, should not affect the source and load terminations for maximum power transfer.

The small signal h-parameters for four transistors were measured over a frequency range extending from one kilocycle to one megacycle. Appendix A contains a discussion of the method used for measuring the h-parameters and a tabulation of the data obtained. With the data from Appendix A, equations 13 and 14 were used to calculate the conjugate-image immittances. The results for Transistors 1, 3 and 4 are tabulated in Appendix B. Transistor 2 was not stable for conjugate-image operation and will be discussed further in the section on stability.

B. Power Gain

Power gain can be defined in several ways for transistor circuits. Three ways of considering power gain are as follows: the ratio of the power output to the power input to the transistor; the ratio of the power output to the power generated within the signal source; the ratio of the power output to the maximum available input power from the signal source. For conjugate-image operation the first and third listed power gains will be the same and the second will be three decibels lower. The power gain considered in this thesis will be the ratio of the power output to the power input to the transistor, which will be the same as the available power gain when properly terminated for conjugate-image operation.

If the network N in Figure 3 is replaced by its h-parameter equivalent circuit, the following equations can be written

$$V_g = I_1(h_{11} + Z_{c1}) + V_2h_{12} \qquad 22$$

and

$$0 = I_1h_{21} + V_2(h_{22} + Y_{c2}). \qquad 23$$

Solving for I_1 and V_2 and letting

$$\triangle = (h_{11} + Z_{c1})(h_{22} + Y_{c2}) - h_{12}h_{21} \tag{24}$$

gives

$$I_1 = \frac{V_g(h_{22} + Y_{c2})}{\triangle} \tag{25}$$

and

$$V_2 = \frac{V_g h_{21}}{\triangle}. \tag{26}$$

The input power will be

$$P_1 = \left|I_1\right|^2 \mathcal{Re}(Z_{c1}) = \left|\frac{V_g(h_{22} + Y_{c2})}{\triangle}\right|^2 r_{11}\phi_r \tag{27}$$

and the output power will be

$$P_o = \left|V_2\right|^2 \mathcal{Re}(Y_{c2}) = \left|\frac{V_g h_{21}}{\triangle}\right|^2 g_{22}\phi_g. \tag{28}$$

The power gain will be

$$G_{pm} = \frac{P_o}{P_1} = \frac{\left|h_{21}\right|^2 g_{22}}{\left|h_{22} + Y_{c2}\right|^2 r_{11}} \tag{29}$$

which reduces to

$$G_{pm} = \frac{\left|h_{21}\right|^2}{r_{11}g_{22}\left(1 + 2\phi_r + \phi_r^2 + \phi_x^2\right)}. \tag{30}$$

For a neutralized network $\phi_x = 0$ and $\phi_r = 1$, so that

$$G_{pn} = \frac{\left|h_{21}\right|^2}{4r_{11}g_{22}}. \tag{31}$$

The data from Appendices A and B were used to calculate the conjugate-image matched power gain. The results are tabulated in Appendix C, Table 13. The calculated power gain assuming neutralization is tabulated in Appendix C, Table 14.

The reverse power gain or the power gain from the normal output terminals 2-2' to the input terminals 1-1' for conjugate-image operation can be determined in manner similar to that used for the forward power gain. For the circuit shown in Figure 4 the following equations can be written

$$0 = (h_{11} + Z_{c1})I_1 + h_{12}V_2 \tag{32}$$

and

$$I_o = h_{21}I_1 + (h_{22} + Y_{c2})V_2. \tag{33}$$

Solving for I_1 and V_2 gives

$$I_1 = \frac{-h_{12}I_o}{\triangle} \tag{34}$$

and

$$V_2 = \frac{(h_{11} + Z_{c1})I_o}{\triangle} \tag{35}$$

where

$$\triangle = (h_{11} + Z_{c1})(h_{22} + Y_{c2}) - h_{12}h_{21}. \tag{36}$$

The power supplied to Z_{c1} will be

$$P_{or} = \left|I_1\right|^2 r_{11}\phi_r \tag{37}$$

and the power into terminals 2-2' will be

$$P_{ir} = \left|V_2\right|^2 g_{22}\phi_r. \tag{38}$$

Then the reverse power gain will be

$$G_{pr} = \frac{P_{or}}{P_{ir}} = \frac{\left|I_1\right|^2 r_{11}\phi_r}{\left|V_2\right|^2 g_{22}\phi_r} \tag{39}$$

or

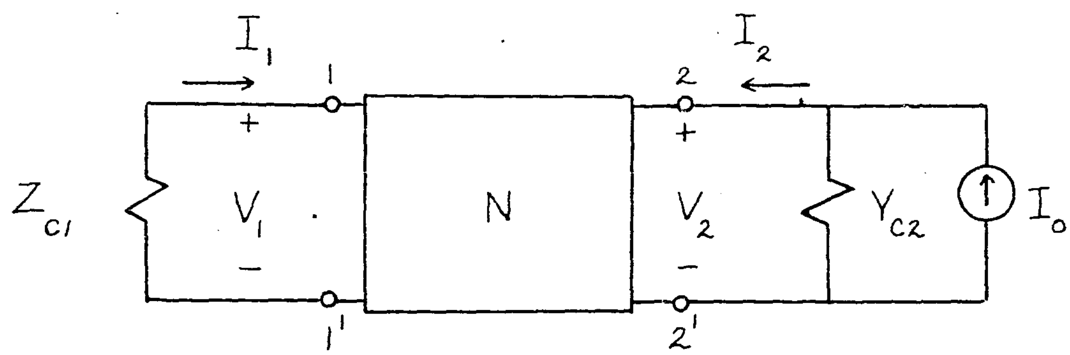


Figure 4. A two terminal pair network connected for reverse transmission

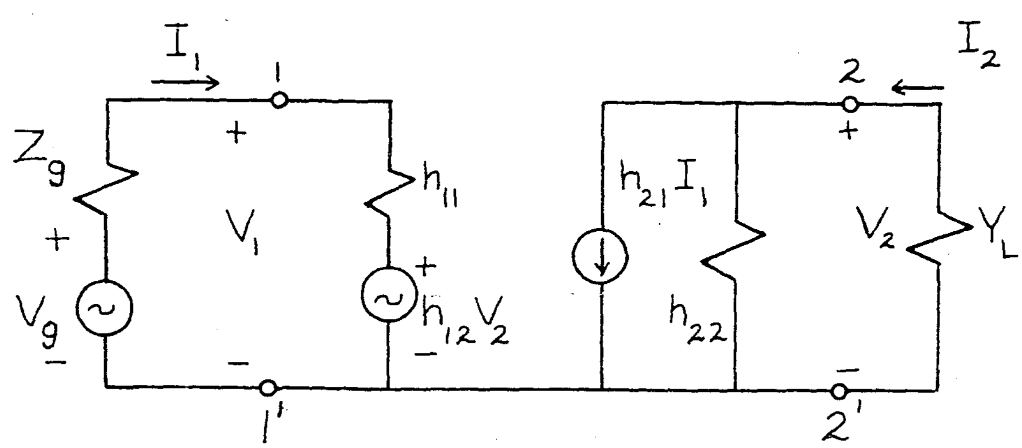


Figure 5. A two terminal pair network with general source and load terminations

$$G_{pr} = \frac{\left|h_{12}\right|^2}{r_{11}g_{22}(1 + 2\phi_r + \phi_r^{\,2} + \phi_x^{\,2})}. \qquad 40$$

This power gain will be small since h_{12} is small. For the transistors tested h_{12e} was of the order of 10^{-4}, while h_{21e} was around 10. The subscript e on h-parameter terms is used to indicate common-emitter operation of the transistor. These values of h_{12e} and h_{21e} indicate a reverse power gain in the order of 10^{-10} times the forward power gain or approximately -60 decibels.

Consider the case of an amplifier that is not properly terminated for conjugate-image operation, as is shown in Figure 5, then

$$V_g = (h_{11} + Z_g)I_1 + h_{12}V_2 \qquad 41$$

and

$$0 = h_{21}I_1 + (h_{22} + Y_L)V_2. \qquad 42$$

Let

$$\rho_1 = \frac{Z_g - Z_{c1}}{Z_g + Z_{c1}^{\,*}} \qquad 43$$

and

$$\rho_2 = \frac{Y_L - Y_{c2}}{Y_L + Y_{c2}^{\,*}}. \qquad 44$$

Where p_1 and p_2 are the input and load reflection factors respectively, solving for I_1 and V_2 gives

$$I_1 = \frac{h_{22} + Y_L}{\triangle} V_g \qquad 45$$

and

$$V_2 = \frac{-h_{21}V_g}{\triangle} \qquad 46$$

where

$$\triangle = (h_{22} + Y_L)(h_{11} + Z_g) - h_{12}h_{21}. \qquad 47$$

Then the power gain will be

$$G_p = \frac{|V_2|^2 \mathcal{R}\ell(Y_L)}{|I_1|^2 \mathcal{R}\ell(Z_{in})} \qquad 48$$

$$G_p = \frac{|h_{21}|^2 \mathcal{R}\ell(Y_L)}{|h_{22} + Y_1|^2 \mathcal{R}\ell(r_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L})}. \qquad 49$$

By defining

$$\rho_c = \frac{h_{11} - Z_{c1}^*}{h_{11} + Z_{c1}} = \frac{h_{22} - Y_{c2}^*}{h_{22} + Y_{c2}} \qquad 50$$

and performing some algebraic manipulations it can be shown that

$$\mathcal{R}\ell(Y_L) = g_{22}\phi_r \frac{1 - \rho_2\rho_2^*}{(1 - \rho_2)(1 - \rho_2^*)} \qquad 51$$

$$h_{22} + Y_L = (h_{22} + Y_{c2}) \frac{1 - \rho_2\rho_c}{1 - \rho_2} \qquad 52$$

$$Z_{in} = \frac{Z_{c1}^* + Z_{c1}\rho_2\rho_c}{1 - \rho_2\rho_c} \qquad 53$$

and

$$G_p = G_{pm} \frac{1 - \rho_2\rho_2^*}{1 - \rho_2\rho_2^*\rho_c\rho_c^*} \qquad 54$$

where G_{pm} is the power gain for conjugate-image operation. This gain, as expected will be less than the matched power gain, since $|\rho_2| < 1$ and

$|\rho_c| < 1$ for $\mathcal{Re}(Y_L) < 0$ and $\phi_r < 0$.

C. Stability

Junction transistors are quite often unstable over a range of frequencies for particular source and load impedances. These transistors are said to be potentially unstable. However, a stable amplifier can be constructed to operate in a frequency range where a transistor is potentially unstable by using the proper source and load impedances. Generally in designing commercial amplifiers, where the transistors in the amplifier might have to be replaced, a factor of safety is used to insure stability.

If a transistor is potentially unstable for a range of frequencies, it will not be possible to calculate the conjugate-image immittances in that frequency range, since the maximum power gain will be infinite. A neutralized amplifier will be stable over the entire frequency for any passive termination.

The following two equations can be written for the circuit shown in Figure 5

$$V_g = (h_{11} + Z_g)\, I_1 + h_{12}V_2 \tag{55}$$

$$0 = h_{21}I_1 + (h_{22} + Y_L)\, V_2. \tag{56}$$

Solving these equations for I_1 gives

$$I_1 = \frac{(h_{22} + Y_L)\, V_g}{(h_{11} + Z_g)(h_{22} + Y_L) - h_{12}h_{21}} \tag{57}$$

or

$$I_1 = \frac{V_g}{h_{11} + Z_g - \frac{h_{12}h_{21}}{h_{22} + Y_L}}. \qquad 58$$

The voltage V_1 at the input to the transistor will be

$$V_1 = V_g - I_1 Z_g \qquad 59$$

or

$$V_1 = V_g \left(\frac{h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L}}{h_{11} + Z_g - \frac{h_{12}h_{21}}{h_{22} + Y_L}} \right). \qquad 60$$

For stable operation power must flow from the source into the transistor. This requires that numerator of V_1 in equation 60 have a positive real part, that is

$$\mathcal{Re}\left(h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L}\right) > 0. \qquad 61$$

The poorest load condition for stability will occur when $\mathcal{Re}\left(\frac{h_{12}h_{21}}{h_{22} + Y_L}\right)$ is a maximum. This will be when $G_L = 0$ and $B_L = \frac{q_c}{p_c} g_{22} - b_{22}$. Using these values for G_L and B_L equation 61 becomes

$$\mathcal{Re}\left(r_{11} + jx_{11} - \frac{p_c^2 - q_c^2 + j2p_cq_c}{g_{22} + j\frac{q_c}{p_c} g_{22}}\right) > 0 \qquad 62$$

or

$$g_{22}r_{11} - p_c^2 > 0. \qquad 63$$

This result indicates that the transistor is potentially unstable when

$$p_c^2 \geqslant g_{22}r_{11} \qquad 64$$

or

$$\left(\mathcal{R}e\left(\sqrt{h_{12}h_{21}}\right)\right)^2 > g_{22}r_{11}. \qquad 65$$

Consider the conjugate-image termination, then

$$\mathcal{R}e\left(r_{11} + jx_{11} - \frac{g_{22}r_{11}\left[(1 - \phi_r^{\ 2} - \phi_x^{\ 2}) + j2\phi_x\right]}{g_{22}(1 + \phi_r + j\phi_x)}\right) > 0 \qquad 66$$

or

$$\mathcal{R}e(r_{11} + jx_{11} - r_{11}(1 - \phi_r + j\phi_x)) > 0 \qquad 67$$

which reduces to

$$\phi_r > 0. \qquad 68$$

Consider equation 64 at the boundary of the instability region, where $p_c^{\ 2} = r_{11}g_{22}$. Using this value of p_c in equation 16 gives

$$\phi_r = \sqrt{1 - \frac{q_c^{\ 2}}{r_{11}q_{22}} - \frac{r_{11}q_{22} - q_c^{\ 2}}{r_{11}q_{22}}} = 0. \qquad 69$$

This result indicates that the boundary for potential instability of a transistor is the same as the boundary for unstable operation for conjugate-image termination.

Of the four transistors whose small-signal parameters were measured only one was unstable for conjugate-image operation. This was Transistor 2. A study of the data indicates that it is potentially unstable from a frequency of about 10,000 to 39,000 cycles per second.

D. Transmission Matrix

The transmission matrix of a two-terminal pair network relates the input current and voltage to the output current and voltage. It is of importance in the cascade operation of networks. For two networks connected in cascade the overall transmission matrix will be the product of the individual transmission matrices. The transmission matrix is usually expressed in terms of ABCD parameters as follows

$$\begin{Vmatrix} V_1 \\ \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A & B \\ \\ C & D \end{Vmatrix} \begin{Vmatrix} V_2 \\ \\ -I_2 \end{Vmatrix} . \qquad 70$$

In this section the transmission matrix will be expressed in terms of the conjugate-image immittances and the forward and reverse transmission functions.

Literature dealing with the matrix analysis of two-terminal pair networks is extensive (5)(20)(33)(34)(35). Several authors (1)(2)(28)(39) treat the iterative operation of cascade networks. This involves raising the transmission matrix to a power n, where n is the number of stages. Pipes (28) expresses the transmission matrix in terms of the iterative impedances, however the case considered is a passive network. Knausenberger (21) expresses the transmission matrix in terms of the image impedances for an active two terminal pair network.

The transmission matrix can be expressed in terms of h-parameters as follows

$$\begin{Vmatrix} A & B \\ \\ C & D \end{Vmatrix} = \begin{Vmatrix} -\dfrac{h}{h_{21}} & -\dfrac{h_{11}}{h_{21}} \\ \\ -\dfrac{h_{22}}{h_{21}} & -\dfrac{1}{h_{21}} \end{Vmatrix} . \qquad 71$$

The forward transfer function will be defined as

$$\theta_1 = \frac{1}{2} \ln \left(\frac{V_1 I_1}{V_2(-I_2)}\right) \qquad 72$$

and the reverse transfer function will be defined as

$$\theta_2 = \frac{1}{2} \ln \left(\frac{V_2 I_2}{V_1(-I_1)}\right). \tag{73}$$

Using these definitions

$$e^{\theta_1} = -\frac{I_1}{I_2}\sqrt{Z_{c1}^{*}Y_{c2}} = +\frac{V_1}{V_2}\frac{1}{\sqrt{Z_{c1}^{*}Y_{c2}}} \tag{74}$$

and

$$e^{\theta_2} = -\frac{I_2}{I_1}\frac{1}{\sqrt{Z_{c1}Y_{c2}^{*}}} = \frac{V_2}{V_1}\sqrt{Z_{c1}Y_{c2}^{*}}. \tag{75}$$

The transmission functions can be expressed as a function of the h-parameters and the conjugate-image immittances as follows

$$e^{\theta_1} = -\frac{h_{22} + Y_{c2}}{h_{21}}\sqrt{\frac{Z_{c1}}{Y_{c2}}^{*}} \tag{76}$$

or

$$e^{\theta_1} = \frac{+h_{12}}{Z_{c1}^{*} - h_{11}}\sqrt{\frac{Z_{c1}}{Y_{c2}}^{*}} \tag{77}$$

and

$$e^{\theta_2} = \frac{h_{11} + Z_{c1}}{h_{12}}\sqrt{\frac{Y_{c2}}{Z_{c1}}^{*}} \tag{78}$$

or

$$e^{\theta_2} = \frac{-h_{21}}{Y_{c2}^{*} - h_{22}}\sqrt{\frac{Y_{c2}}{Z_{c1}}^{*}}. \tag{79}$$

Equations 76 and 79 involve only h_{21}, h_{22}, conjugate-image immittances and the transmission functions. These two equations can be used to obtain

$$h_{21} = \frac{-\sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}}\,(Y_{c2} + Y_{c2}^{*})}{+\,e^{\theta_1} + e^{-\theta_2}\sqrt{\dfrac{Z_{c1}^{*}Y_{c2}^{*}}{Z_{c1}Y_{c2}}}} \qquad 80$$

and

$$h_{22} = \frac{Y_{c2}^{*}\left(+\,e^{\theta_1} - e^{-\theta_2}\sqrt{\dfrac{Y_{c2}Z_{c1}^{*}}{Z_{c1}Y_{c2}^{*}}}\right)}{e^{\theta_1} + e^{-\theta_2}\sqrt{\dfrac{Z_{c1}^{*}Y_{c2}^{*}}{Z_{c1}Y_{c2}}}}. \qquad 81$$

Similarly equations 77 and 78 can be used to obtain

$$h_{11} = \frac{Z_{c1}^{*}\left(e^{\theta_1} - e^{-\theta_2}\sqrt{\dfrac{Z_{c1}Y_{c2}^{*}}{Z_{c1}^{*}Y_{c2}}}\right)}{e^{\theta_1} + e^{-\theta_2}\sqrt{\dfrac{Z_{c1}^{*}Y_{c2}^{*}}{Z_{c1}Y_{c2}}}} \qquad 82$$

and

$$h_{12} = \frac{e^{\theta_1}\,e^{-\theta_2}\sqrt{\dfrac{Y_{c2}^{*}}{Z_{c1}}}\,(Z_{c1} + Z_{c1}^{*})}{e^{\theta_1} + e^{-\theta_2}\sqrt{\dfrac{Z_{c1}^{*}Y_{c2}^{*}}{Z_{c1}Y_{c2}}}}. \qquad 83$$

The values of the h-parameters in equations 80, 81, 83 and 84 can be substituted into the h-parameter transmission matrix expression of equation 71. This gives the following transmission matrix components

$$A = \frac{Y_{c2}^{*}Z_{c1}^{*}\sqrt{\dfrac{Y_{c2}}{Z_{c1}^{*}}}\,e^{\theta_1} + Y_{c2}Z_{c1}\sqrt{\dfrac{Y_{c2}^{*}}{Z_{c1}}}\,e^{-\theta_2}}{(Y_{c2} + Y_{c2}^{*})} \qquad 84$$

$$B = \frac{Z_{c1}^{*}\sqrt{\frac{Y_{c2}}{Z_{c1}^{*}}}\, e^{\theta_1} - Z_{c1}\sqrt{\frac{Y_{c2}^{*}}{Z_{c1}}}\, e^{-\theta_2}}{(Y_{c2} + Y_{c2}^{*})} \qquad 85$$

$$C = \frac{Y_{c2}^{*}\sqrt{\frac{Y_{c2}}{Z_{c1}^{*}}}\, e^{\theta_1} - Y_{c2}\sqrt{\frac{Y_{c2}^{*}}{Z_{c1}}}\, e^{-\theta_2}}{(Y_{c2} + Y_{c2}^{*})} \qquad 86$$

and

$$D = \frac{\sqrt{\frac{Y_{c2}}{Z_{c1}^{*}}}\, e^{\theta_1} + \sqrt{\frac{Y_{c2}^{*}}{Z_{c1}}}\, e^{-\theta_2}}{(Y_{c2} + Y_{c2}^{*})}. \qquad 87$$

This transmission matrix can be factored into four matrices and scalar multiplier as follows

$$\begin{Vmatrix} Z_{c1}^{*} & Z_{c1} \\ 1 & -1 \end{Vmatrix} \begin{Vmatrix} e^{\theta_1} & 0 \\ 0 & e^{-\theta_2} \end{Vmatrix} \begin{Vmatrix} \sqrt{\frac{Y_{c2}}{Z_{c1}^{*}}} & 0 \\ 0 & \sqrt{\frac{Y_{c2}^{*}}{Z_{c1}}} \end{Vmatrix} \mathrm{X}$$

$$\begin{Vmatrix} Y_{c2}^{*} & 1 \\ Y_{c2} & -1 \end{Vmatrix} \quad \frac{1}{Y_{c2} + Y_{c2}^{*}}. \qquad 88$$

Starting at the left they can be thought of as an input matching matrix, a transmission matrix, a matrix for changing impedance level and an output matching matrix respectively.

For a cascade of M networks that are conjugate-image matched the transmission matrix will be

$$\begin{Vmatrix} {}_1Z_{c1}^{*} & {}_1Z_{c1} \\ 1 & -1 \end{Vmatrix} \begin{Vmatrix} e^{\Sigma \theta_{1n}} & 0 \\ 0 & e^{-\Sigma \theta_{2n}} \end{Vmatrix} \begin{Vmatrix} \sqrt{\dfrac{{}_MY_{c2}}{{}_1Z_{c1}^{*}}} & 0 \\ 0 & \sqrt{\dfrac{{}_MY_{c2}^{*}}{{}_1Z_{c1}}} \end{Vmatrix} \times$$

$$\begin{Vmatrix} {}_MY_{c2}^{*} & 1 \\ {}_MY_{c2} & -1 \end{Vmatrix} \qquad \frac{1}{{}_MY_{c2} + {}_MY_{c2}^{*}}. \qquad 89$$

The second subscript on the transmission function is used to denote the particular network in the cascade connection, that is θ_{1n} would be the forward transmission function of the n^{th} network numbered from the input. The subscripts ahead of the conjugate-image immittances are used to indicate the network to which they refer.

The output current and voltage can be expressed in terms of the input current and voltage as follows

$$\begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} \dfrac{D}{\Delta} & \dfrac{B}{\Delta} \\ \dfrac{C}{\Delta} & \dfrac{A}{\Delta} \end{Vmatrix} \begin{Vmatrix} V_1 \\ -I_1 \end{Vmatrix} \qquad 90$$

where

$$\Delta = A\,D - B\,C. \qquad 91$$

The matrix relating V_2, I_2 and V_1, $(-I_1)$ is the inverse of the transmission matrix with the signs on the 12 and 21 terms reversed. These reversals in signs are caused by changing the reference direction of the input and output current. This matrix will be referred to as the reverse transmission matrix.

The reverse transmission matrix components can be expressed as follows

$$\frac{D}{\Delta} = \frac{\sqrt{\dfrac{Z_{c1}}{Y_{c2}^{*}}}\, e^{\theta_2} + \sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}}\, e^{-\theta_1}}{Z_{c1} + Z_{c1}^{*}} \qquad 92$$

$$\frac{B}{\Delta} = \frac{Z_{c1}^{*}\sqrt{\dfrac{Z_{c1}}{Y_{c2}^{*}}}\, e^{\theta_2} - Z_{c1}\sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}}\, e^{-\theta_1}}{Z_{c1} + Z_{c1}^{*}} \qquad 93$$

$$\frac{C}{\Delta} = \frac{Y_{c2}^{*}\sqrt{\dfrac{Z_{c1}}{Y_{c2}^{*}}}\, e^{\theta_2} - Y_{c2}\sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}}\, e^{-\theta_1}}{Z_{c1} + Z_{c1}^{*}} \qquad 94$$

and

$$\frac{A}{\Delta} = \frac{Y_{c2}^{*}\, Z_{c1}^{*}\sqrt{\dfrac{Z_{c1}}{Y_{c2}^{*}}}\, e^{\theta_2} + Y_{c2}\, Z_{c1}\sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}}\, e^{-\theta_1}}{Z_{c1} + Z_{c1}^{*}}. \qquad 95$$

The reverse transmission matrix can be factored as follows

$$\begin{Vmatrix} 1 & -1 \\ Y_{c2}^{*} & Y_{c2} \end{Vmatrix} \begin{Vmatrix} e^{\theta_2} & 0 \\ 0 & e^{-\theta_1} \end{Vmatrix} \begin{Vmatrix} \sqrt{\dfrac{Z_{c1}}{Y_{c2}^{*}}} & 0 \\ 0 & \sqrt{\dfrac{Z_{c1}^{*}}{Y_{c2}}} \end{Vmatrix} \mathrm{X}$$

$$\begin{Vmatrix} 1 & Z_{c1}^{*} \\ -1 & Z_{c1} \end{Vmatrix} \frac{1}{Z_{c1} + Z_{c1}^{*}}. \qquad 96$$

A cascade connection of M conjugate-image matched networks will have an overall reverse transmission matrix as follows

$$\begin{Vmatrix} 1 & -1 \\ {}_{M}Y_{c2}^{*} & {}_{M}Y_{c2} \end{Vmatrix} \begin{Vmatrix} e^{\Sigma\theta_{2m}} & 0 \\ 0 & e^{-\Sigma\theta_{1m}} \end{Vmatrix} \begin{Vmatrix} \sqrt{\dfrac{{}_{1}Z_{c1}}{{}_{M}Y_{c2}^{*}}} & 0 \\ 0 & \sqrt{\dfrac{{}_{1}Z_{c1}^{*}}{{}_{M}Y_{c2}}} \end{Vmatrix} \mathrm{X}$$

$$\begin{Vmatrix} 1 & {}_{1}Z_{c1}^{*} \\ -1 & {}_{1}Z_{c1} \end{Vmatrix} \frac{1}{{}_{1}Z_{c1} + {}_{1}Z_{c1}^{*}} \,. \qquad 97$$

III. THE APPROXIMATION OF CONJUGATE-IMAGE IMMITTANCES

A. General Discussion

The approximation problem considered in this section involves the representation of conjugate-image immittances as the ratio of two polynomials of frequency. The numerator and denominator polynomials will be of the same order or differ at most by one. Allowing W to represent an immittance, it can be expressed as follows

$$W = \frac{A_m p^m + A_{m-1} p^{m-1} + \text{------------} + A_1 p + A_o}{p^n + B_{n-1} p^{n-1} + \text{------------} + B_1 p + B_o} \qquad 98$$

where $p = j\omega$ for real frequencies. W can also be expressed in a factored form as

$$W = \frac{A_m (p-a_m)(p-a_{m-1}) \text{ ------------ } (p-a_1)}{(p-b_n)(p-b_{n-1}) \text{ ------------ } (p-b_1)} \qquad 99$$

where the a_k's are the zeros and the b_j's are the poles of W. For a stable passive two terminal network the poles and zeros will be restricted to the left half of the p-plane.

An immittance W whose poles are restricted to the left half plane will be analytic in the right half plane. However, the conjugate of W will not be analytic in the right half plane. In the conjugate-image operation of a network, the input impedance and its conjugate are considered, so one or both of these impedances will not be analytic. A similar statement can be made about the output admittance and its conjugate. Therefore, the conjugate-image immittances will generally have poles and zeros in the right half plane if the approximation is to cover a wide range of frequencies.

B. Representation of Conjugate-Image Immittances

Two examples will be used for the approximation and circuit representation of conjugate-image immittances. They are Z_{c1} for Transistor 1 and Y_{c2} for Transistor 3.

Calculated values of Z_{c1} for Transistor 1 are given in Table 11 and plotted in Figure 6. Assume

$$Z_{c1} = \frac{Ap^3 + Bp^2 + Cp + D}{p^3 + Ep^2 + Fp + G}. \qquad 100$$

Letting $Z_{c1} = R_{c1} + jX_{c1}$ and $p = j\omega$ equation 100 can be written

$$R_{c1} + jX_{c1} = \frac{D - \omega^2 B + j(\omega C - \omega^3 A)}{G - \omega^2 E + j(\omega F - \omega^3)}. \qquad 101$$

Multiplying both sides of the equation by $G - \omega^2 E + j(\omega F - \omega^3)$ and equating the real parts and the imaginary parts gives

$$\omega^3 A - \omega C - \omega^2 X_{c1} E + \omega R_{c1} F + X_{c1} G = \omega^3 R_{c1} \qquad 102$$

and

$$\omega^2 B - D - \omega^2 R_{c1} E - \omega X_{c1} F + R_{c1} G = -\omega^3 X_{c1}. \qquad 103$$

The seven unknown polynomial coefficients can be determined by using equations 102 and 103 and choosing values of ω. The values used were $\omega = 0$, $\omega = 10^5$, $\omega = 3 \times 10^5$, $\omega = 10^6$ in equation 103 and $\omega = 5 \times 10^4$, $\omega = 2 \times 10^5$ and $\omega = 5 \times 10^5$ in equation 102. R_{c1} and X_{c1} were determined for these frequencies and substituted into the equations. The seven equations were used to determine the following polynomial coefficients.

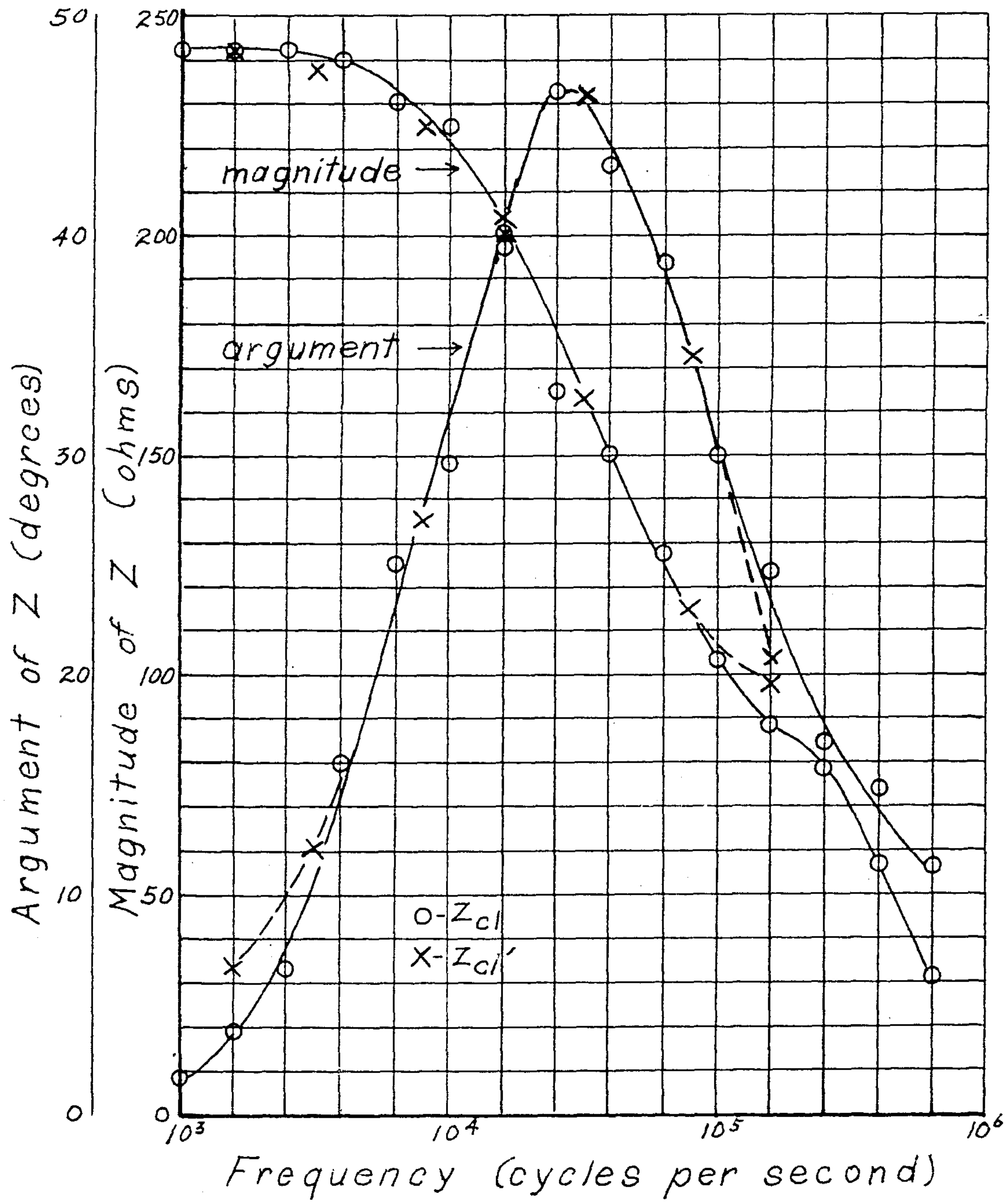


Figure 6. Z_{c1} for Transistor 1

$$A = 90.3 \tag{104}$$

$$B = -4.44 \times 10^{7} \tag{105}$$

$$C = -5.27 \times 10^{11} \tag{106}$$

$$D = 6.39 \times 10^{17} \tag{107}$$

$$E = -9.75 \times 10^{4} \tag{108}$$

$$F = -3.06 \times 10^{10} \tag{109}$$

and

$$G = 2.63 \times 10^{15}. \tag{110}$$

Then

$$Z_{cl}' = \frac{90.3p^{3} - 4.44 \times 10^{7}p^{2} - 5.27 \times 10^{11}p + 6.39 \times 10^{17}}{p^{3} - 9.75 \times 10^{4}p^{2} - 3.06 \times 10^{10}p + 2.63 \times 10^{15}} \tag{111}$$

where the prime is used to indicate that this is an approximation to Z_{cl} and not Z_{cl} itself. The numerator and denominator can be factored to give

$$Z_{cl}' = \frac{90.3(p + 1.127 \times 10^{5})(p - 4.738 \times 10^{5})(p - 1.32 \times 10^{5})}{(p + 1.725 \times 10^{5})(p - 1.867 \times 10^{5})(p - 0.826 \times 10^{5})}. \tag{112}$$

The equation 111 approximation to Z_{cl} is plotted in Figure 6 with Z_{cl}. The magnitude and phase match fairly well up to 10^{6} radians per second, which was the highest frequency used in determining the polynomial coefficients. Using higher order polynomials and larger values of ω for determining the coefficients would give a better match at high frequencies.

Equation 112 indicates two right half plane zeros and two right half plane poles, therefore the circuit representation of Z_{cl}' will have some negative elements. One circuit representation for Z_{cl}' can be obtained from a partial fraction expansion of the impedance function. Such an expansion of Z_{c1}' gives

$$Z_{c1}' = 90.3 - \frac{1.162 \times 10^7}{p + 1.724 \times 10^5} - \frac{1.137 \times 10^7}{p - 1.868 \times 10^5} - \frac{1.282 \times 10^7}{p - 8.26 \times 10^4}. \qquad 113$$

The circuit determined from equation 113 is shown in Figure 7 (A). Another approach would be to make a continued fraction expansion, which in one form gives

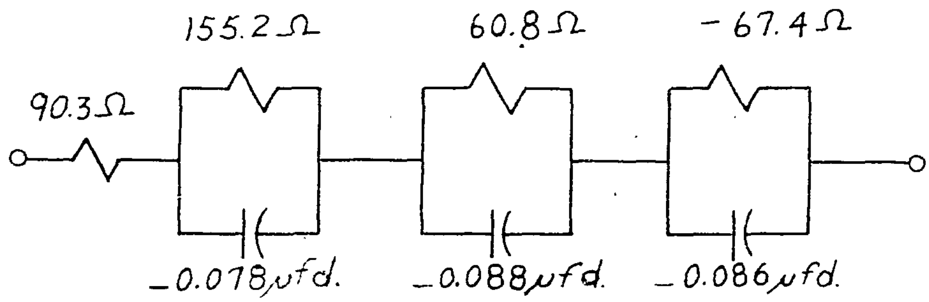
$$Z_{c1}' = 90.3 + \cfrac{1}{-2.81 \times 10^{-8}p + \cfrac{1}{1027 + \cfrac{1}{-1.57 \times 10^{-9}p + \cfrac{1}{-962 + \cfrac{1}{-9.83 \times 10^{-8}p + \cfrac{1}{88.5}}}}}}. \qquad 114$$

The circuit obtained from the continued fraction expansion is shown in Figure 7 (B). These are two of many possible circuit representations for Z_{c1}'.

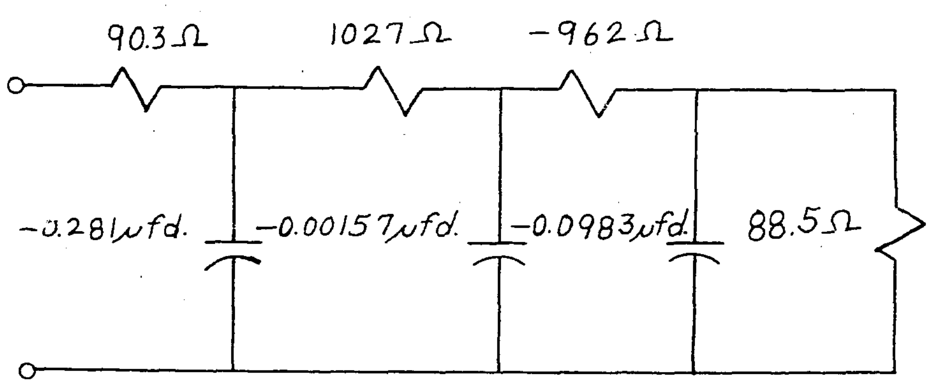
The calculated values of Y_{c2} for Transistor 3 are given in Table 12 and plotted in Figure 8. The method used to approximate Y_{c2} was the same as that used for Z_{c1}. Assume

$$Y_{c2} = \frac{Ap^3 + Bp^2 + Cp + D}{p^2 + Ep + F}. \qquad 115$$

Here the numerator was assumed larger than the denominator, because of the shape of the Y_{c2} curves of magnitude and phase. The six equations needed



(A)



(B)

Figure 7. Circuit representations for Z_{c1}'

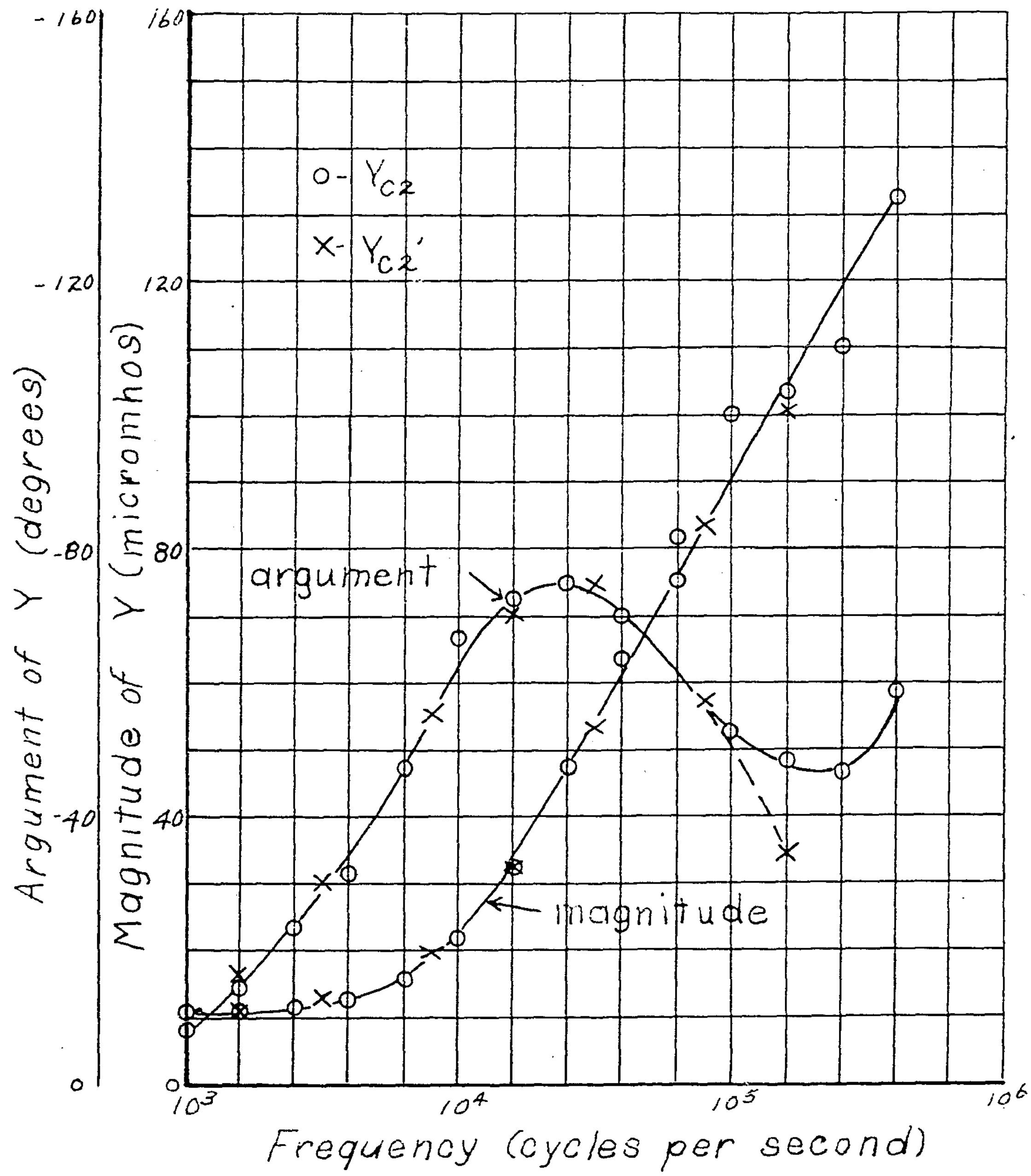


Figure 8. Y_{c2} **for Transistor 3**

to determine the polynomial coefficients can be obtained in the same way as for Z_{c1}. In solving the equations A was found to be negligible and

$$B = 1.11 \times 10^{-4} \tag{116}$$

$$C = 35.2 \tag{117}$$

$$D = -1.348 \times 10^{6} \tag{118}$$

$$E = -3.51 \times 10^{5} \tag{119}$$

and

$$F = -1.17 \times 10^{11}. \tag{120}$$

Thus

$$Y_{c2}' = \frac{1.11 \times 10^{-4} p^2 + 35.2p - 1.348 \times 10^6}{p^2 - 3.51 \times 10^5 p - 1.17 \times 10^{11}} \tag{121}$$

and factoring the numerator and denominator gives

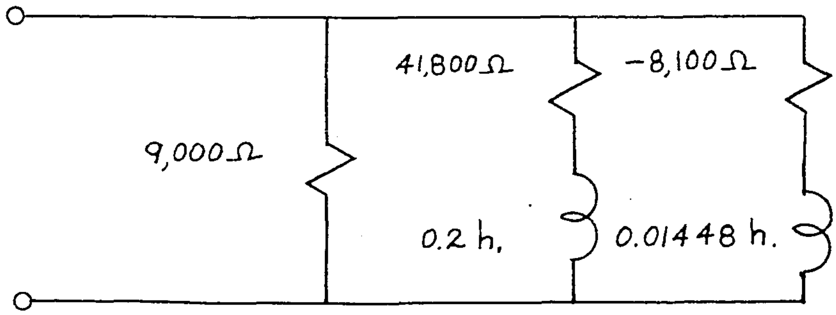
$$Y_{c2}' = \frac{1.11 \times 10^{-4}(p + 3.515 \times 10^5)(p - 0.345 \times 10^5)}{(p + 2.09 \times 10^5)(p - 5.6 \times 10^5)}. \tag{122}$$

The curves of magnitude and phase for Y_{c2}' match those of Y_{c2} fairly well up to 10^5 cycles per second.

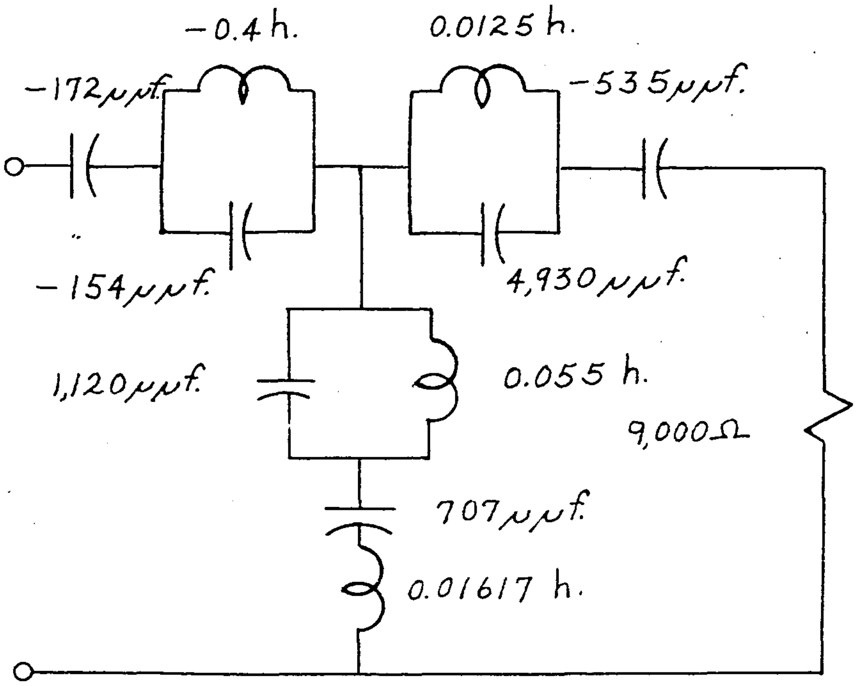
A circuit representation for Y_{c2}' can be determined from a partial fraction expansion of equation 122. An expansion of Y_{c2}' gives

$$Y_{c2}' = 1.11 \times 10^{-4} + \frac{5}{p + 2.09 \times 10^5} + \frac{69.1}{p - 5.6 \times 10^5}. \tag{123}$$

The circuit determined from the partial fraction expansion is shown in Figure 9 (A). The Darlington procedure can be used to obtain a circuit representation that consists of a lossless two terminal pair network terminated in a resistance (19). The network obtained, which is shown in Figure 9 (B), is considerably more complicated than for the partial fraction expansion, however it has the advantage of separating the network



(A)



(B)

Figure 9. Circuit representations for Y_{c2}'

into a lossless portion and one resistor. It is usually desirable to have a single power dissipating element in the load impedance of an amplifier. These networks are not physically realizable, since they involve negative elements.

IV. SINGLE STAGE AMPLIFIERS

A. Conjugate-Image Operation

The single stage power gain for conjugate-image operation of an amplifier was experimentally determined using two different transistors. Transistors 1 and 3 were used. However, Transistor 4 is stable for conjugate-image operation and could have been used in the amplifier. The amplifiers were operated as small signal stages, so the transistor h-parameters could be assumed constant with respect to signal level. The quiescent operating point was made the same as that used to measure the small signal h-parameters, that is $V_c = -4.5$ volts and $I_c = -1$ milliampere.

For conjugate-image operation the output impedance is high, about 90,000 ohms maximum, and the input impedance is low, around 300 ohms. Small signal operation indicates operation over the linear portion of the transistor characteristics, which limits the signal voltage at the collector to one or two volts when the quiescent collector voltage is -4.5 volts. Using one volt as an example, the power level at the output would be about 11 microwatts for a 90,000 ohm load. Since these transistors have power gains of about 40 decibels, the input power will be very small, about 0.0011 microwatts. This means that the input voltage will need to be small, less than one millivolt.

A schematic diagram of the circuit used to experimentally determine the power gain for conjugate-image operation is shown in Figure 10. The collector is series fed through a load impedance equal to the reciprocal of Y_{c2}. In the circuit the source impedance is essentially Z_{c1}, since it

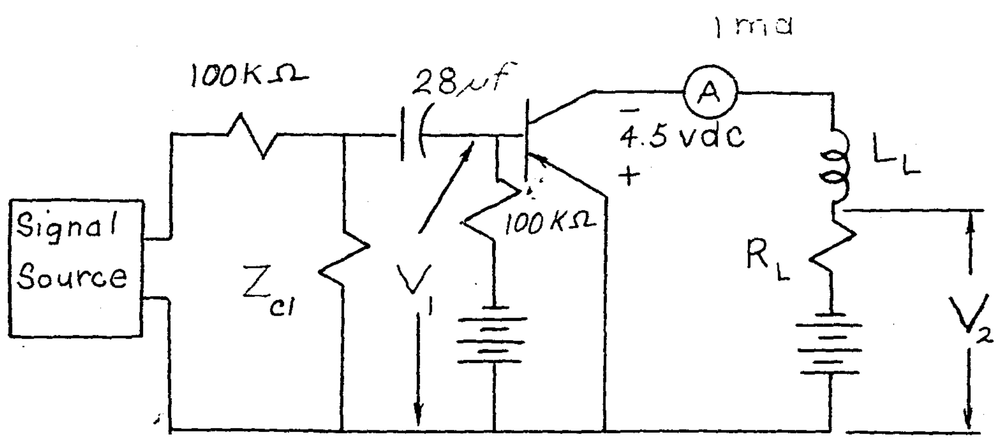


Figure 10. Schematic diagram of the circuit used to measure power gain for conjugate-image operation

is small compared with 100,000 ohms. The source and load impedances were adjusted on a point by point basis, that is for each frequency at which measurements were made the source and load impedances were adjusted to the proper values.

The voltage used to compute the power output, V_2, was measured across the resistive portion of the load impedance. Then the power output will be

$$P_o = \frac{V_2^2 Y_{c2} Y_{c2}^*}{G_{c2}} \qquad 124$$

where G_{c2} is the real part of Y_{c2}. The input voltage was measured from base to emitter, so the power input will be

$$P_{in} = \frac{V_1^2 R_{c1}}{Z_{c1} Z_{c1}^*} \qquad 125$$

and R_{c1} is the real part of Z_{c1}. The results of these measurements are tabulated in Table 1 and plotted in Figures 11 and 12.

The experimental and calculated curves agree fairly well. The leveling off and then increasing of the calculated power gain is probably due to errors in measuring h_{12e} at the higher frequencies. The experimental power gain curve falls off at higher frequencies at the rate of about six decibels per octave, which is the value Drouilhet (12) indicates.

B. Neutralized Amplifier Operation

The small signal parameter h_{12} is called the feedback parameter or reverse transmission parameter. This is the term which supplies signal to the input from the output, as the grid to plate capacitance does in a

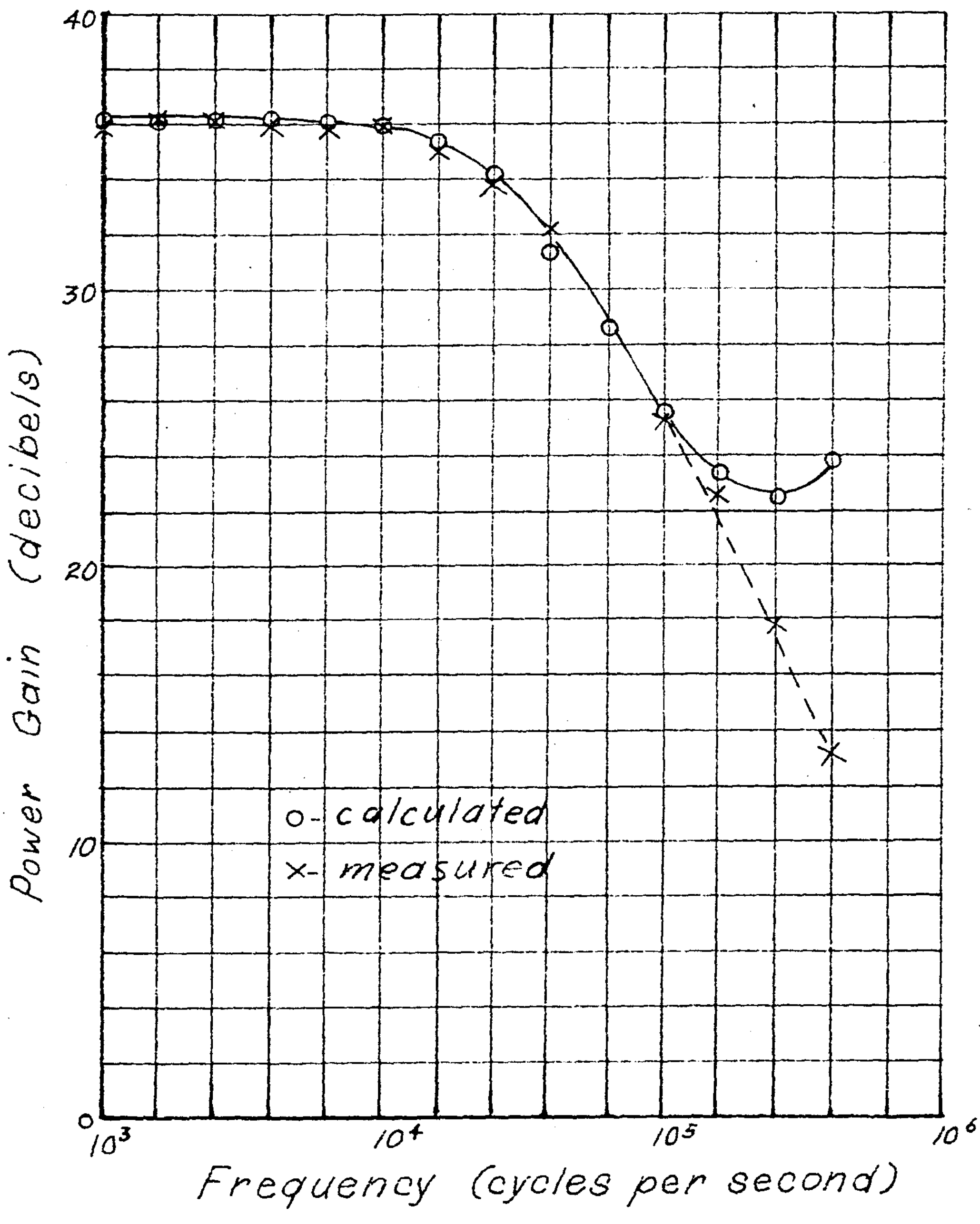


Figure 11. Amplifier power gain for conjugate-image operation using Transistor 1

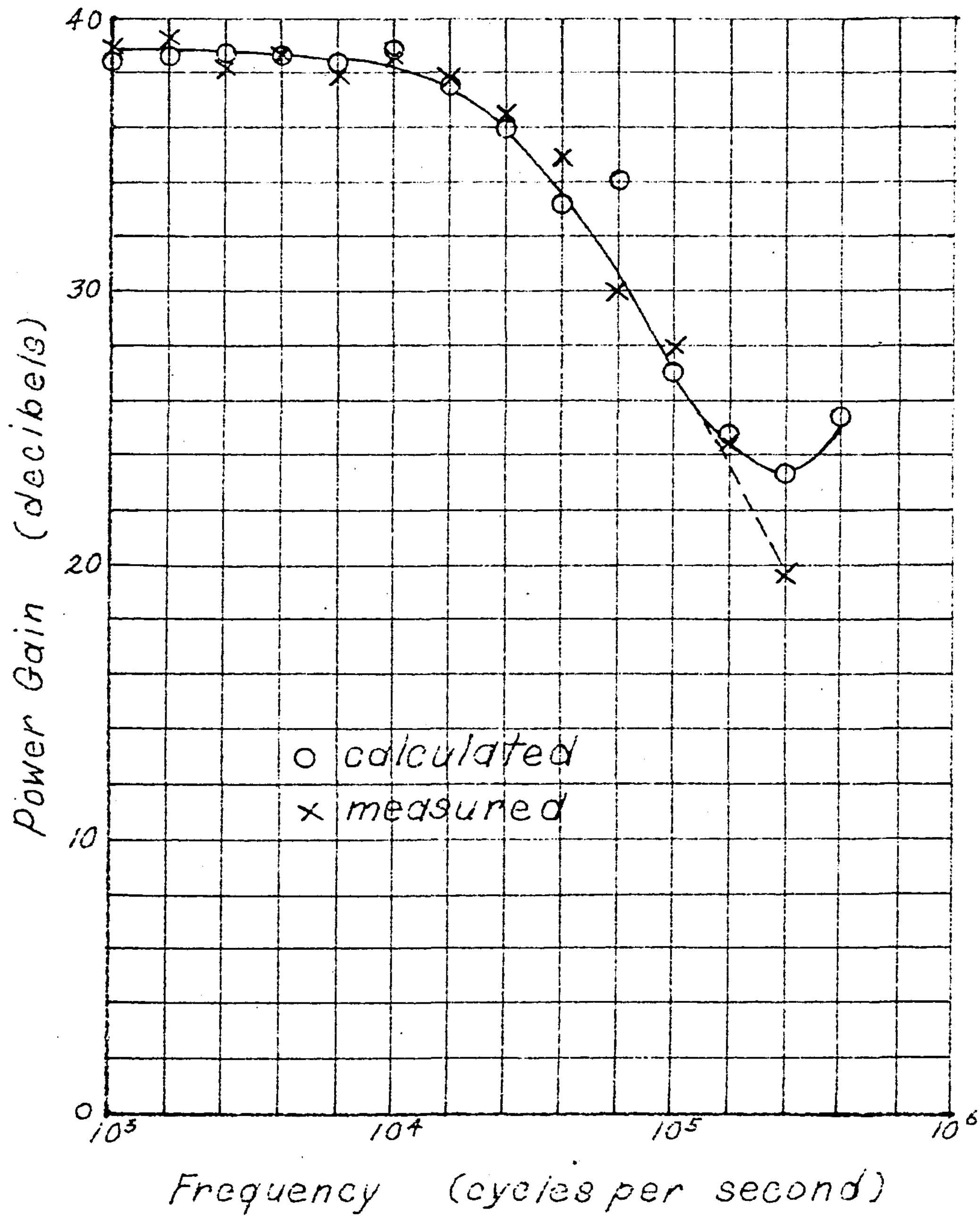


Figure 12. Amplifier power gain for conjugate-image operation using Transistor 3

Table 1. Measured power gain for conjugate-image operation

Frequency cycles per second	Transistor 1 decibels	Transistor 3 decibels
1,000	35.94	38.94
1,585	36.14	39.22
2,510	36.07	38.03
3,980	35.94	38.66
6,310	35.89	37.95
10,000	35.97	38.56
15,850	34.98	37.74
25,100	33.78	36.55
39,800	32.16	34.98
63,100	-----	30.0
100,000	25.27	27.93
158,500	22.63	23.82
251,000	17.82	19.64
398,000	13.06	-----

vacuum tube. Thus it might be possible to cause instability by using the proper terminating impedances. If h_{12} were zero, a transistor amplifier's stability would be independent of the source and load impedances.

A network used to neutralize a transistor will ordinarily affect all of the h-parameter terms. For example consider the circuit shown in Figure 13. This circuit connection satisfies Guillemin's conditions (18) for the addition of the two individual h-parameter matrices to obtain the overall h-parameter matrix. The following matrix equations can be written

$$\begin{Vmatrix} V_1' \\ I_2' \end{Vmatrix} = \begin{Vmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{Vmatrix} \begin{Vmatrix} I_1' \\ V_2' \end{Vmatrix} \qquad 126$$

and

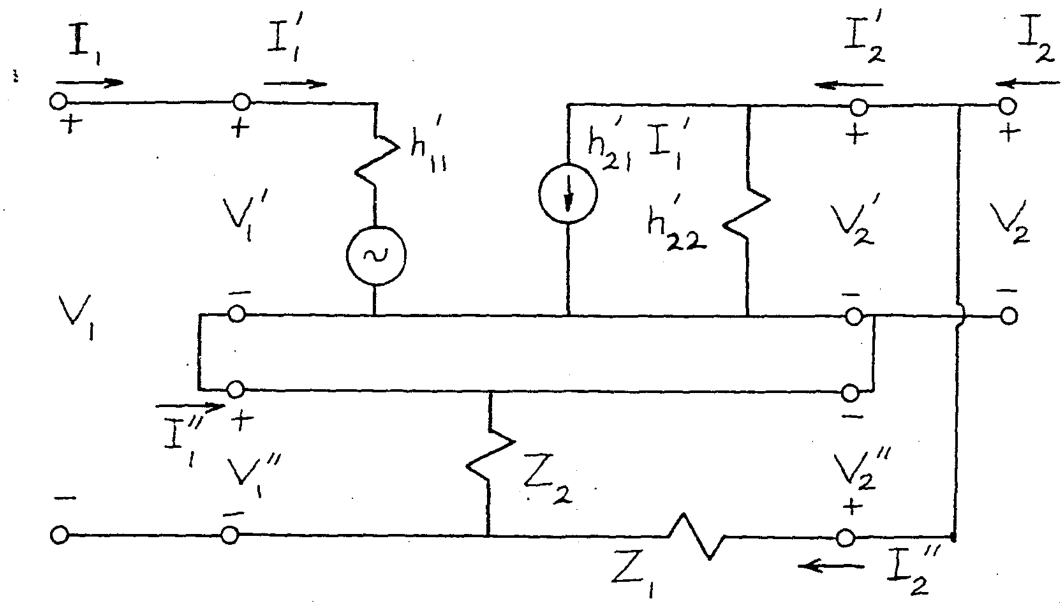


Figure 13. A neutralized two terminal pair network

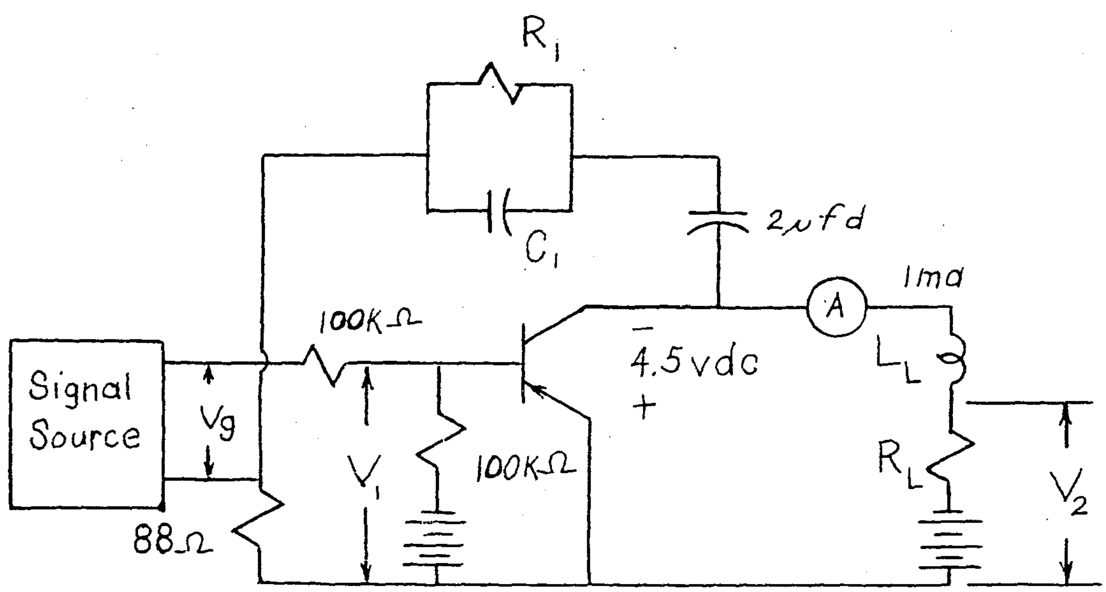


Figure 14. Schematic diagram of the circuit used to measure power gain of a neutralized transistor amplifier

$$\begin{Vmatrix} V_1'' \\ \\ I_2'' \end{Vmatrix} = \begin{Vmatrix} \dfrac{Z_1 Z_2}{Z_1 + Z_2} & - \dfrac{Z_2}{Z_1 + Z_2} \\ \\ \dfrac{Z_2}{Z_1 + Z_2} & \dfrac{1}{Z_1 + Z_2} \end{Vmatrix} \begin{Vmatrix} I_1'' \\ \\ V_2'' \end{Vmatrix}. \qquad 127$$

For the network connected as shown in Figure 13 $V_1 = V_1' + V_1''$, $I_1 = I_1' = I_1''$, $V_2 = V_2' = V_2''$, and $I_2 = I_2' + I_2''$. Adding matrix equations 126 and 127 gives the following matrix equation

$$\begin{Vmatrix} V_1 \\ \\ I_2 \end{Vmatrix} = \begin{Vmatrix} h_{11} + \dfrac{Z_1 Z_2}{Z_1 + Z_2} & h_{12}' - \dfrac{Z_2}{Z_1 + Z_2} \\ \\ h_{21}' + \dfrac{Z_2}{Z_1 + Z_2} & h_{22}' + \dfrac{1}{Z_1 + Z_2} \end{Vmatrix} \begin{Vmatrix} I_1 \\ \\ V_2 \end{Vmatrix}. \qquad 128$$

If the $Z_1 Z_2$ network is to neutralize the transistor, then

$$h_{12} = h_{12}' - \frac{Z_2}{Z_1 + Z_2} = 0 \qquad 129$$

or

$$h_{12}' = \frac{Z_2}{Z_1 + Z_2}. \qquad 130$$

Equation 130 can be written in terms of admittances as follows

$$h_{12}' = \frac{Y_1}{Y_1 + Y_2}. \qquad 131$$

Since h_{12}' is small for transistors, Y_2 can be made large with respect to Y_1 and

$$h_{12}' = \frac{Y_1}{Y_2} \qquad 132$$

or

$$Y_1 = h_{12}' Y_2. \qquad 133$$

Z_2 was made resistive and equal to 88 ohms, then the values for Y_1 were calculated. For the conjugate-image operation of a neutralized amplifier

$$Y_L = h_{22}^* = h_{22}'^* + \frac{1}{Z_1^* + Z_2^*} \qquad 134$$

or since $Z_1 \gg Z_2$

$$Y_L = h_{22}'^* + Y_1^*. \qquad 135$$

The output impedance is independent of the source impedance for a neutralized amplifier, therefore no attempt was made to match the input impedance when measuring the power gain.

The schematic diagram of the circuit used to measure power gain is shown in Figure 14. The voltage, V_2, across R_L was measured and used to calculate the power output

$$P_o = \frac{V_2^2}{R_L}. \qquad 136$$

The signal source was connected in series with a 100 kilohm resistor, which is large compared to the input impedance of the neutralized transistor. Then the power output will be

$$P_{in} = \left(\frac{V_g}{10^5}\right)^2 \mathcal{R}e\left(h_{11}' + \frac{Z_1 Z_2}{Z_1 + Z_2}\right) \qquad 137$$

or

$$P_{in} = \left(\frac{V_g}{10^5}\right)^2 R_{in} \qquad 138$$

where R_{in} is the real part of the input impedance. The power gain of an amplifier for conjugate-image operation can be calculated from

$$G_p = \frac{|h_{21}|^2}{4r_{11}g_{22}} \qquad 139$$

where h_{21}, r_{11} and g_{22} are for the transistor and neutralizing circuit combination.

The calculated power gains for the neutralized transistor amplifiers are tabulated in Table 2 and plotted in Figures 15 and 16. The measured

Table 2. Calculated power gain for neutralized amplifier

Frequency cycles per second	Transistor 1 decibels	Transistor 3 decibels
1,000	34.25	36.95
1,585	34.12	37.06
2,510	34.12	37.14
3,980	33.87	36.99
6,310	33.43	35.86
10,000	32.86	34.08
15,850	31.56	32.22
25,100	29.62	29.48
39,800	26.91	26.34
63,100	24.22	24.84
100,000	21.3	20.30
158,500	19.4	17.78
251,000	18.1	16.35

results are contained in Table 3 and Figures 15 and 16. The experimental and calculated power gains are practically identical up to 10^5 cycles per second for Transistor 1. The deviation between the calculated and experimental power gains was about one decibel at low frequencies for Transistor

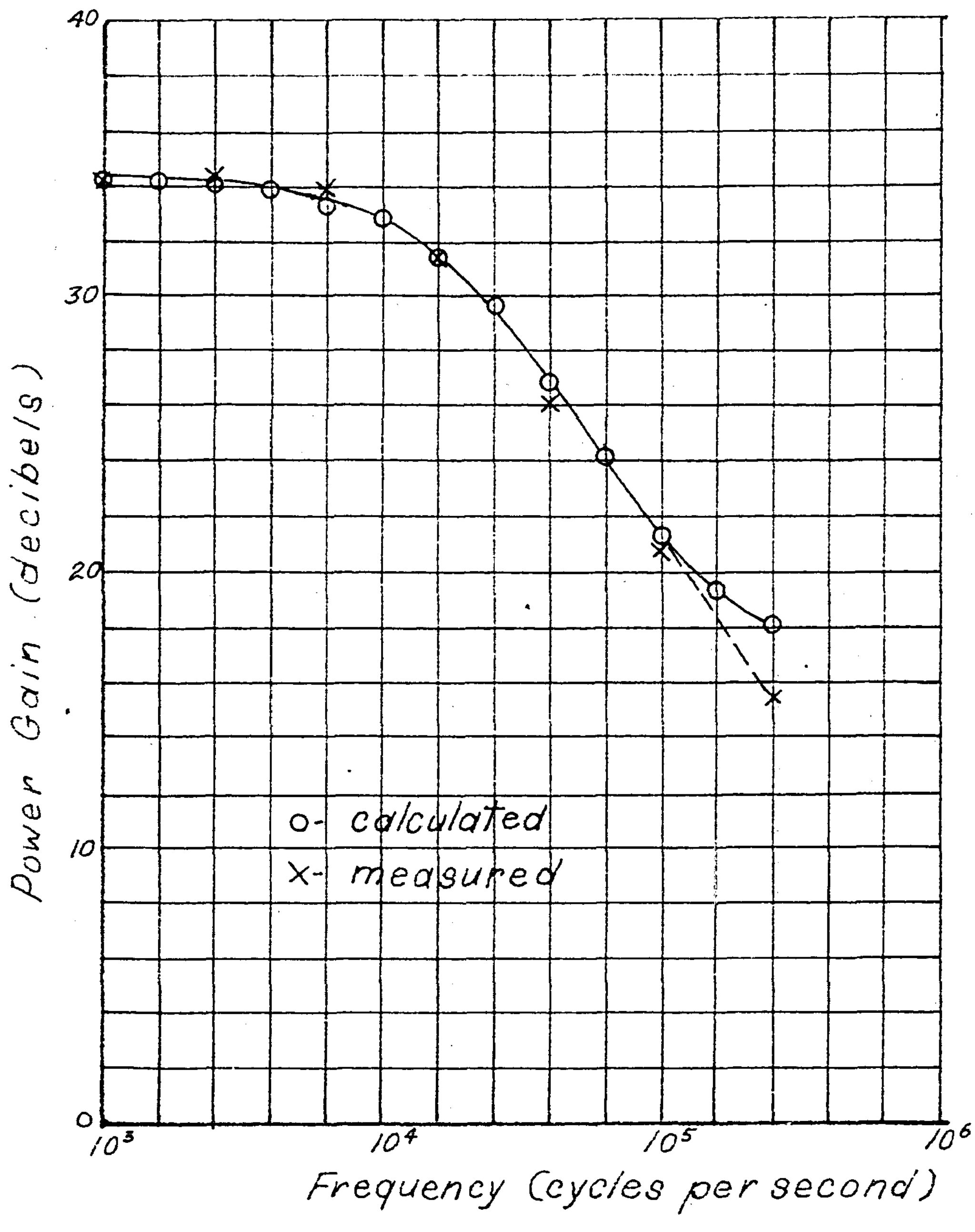


Figure 15. Power gain of neutralized transistor amplifier using Transistor 1

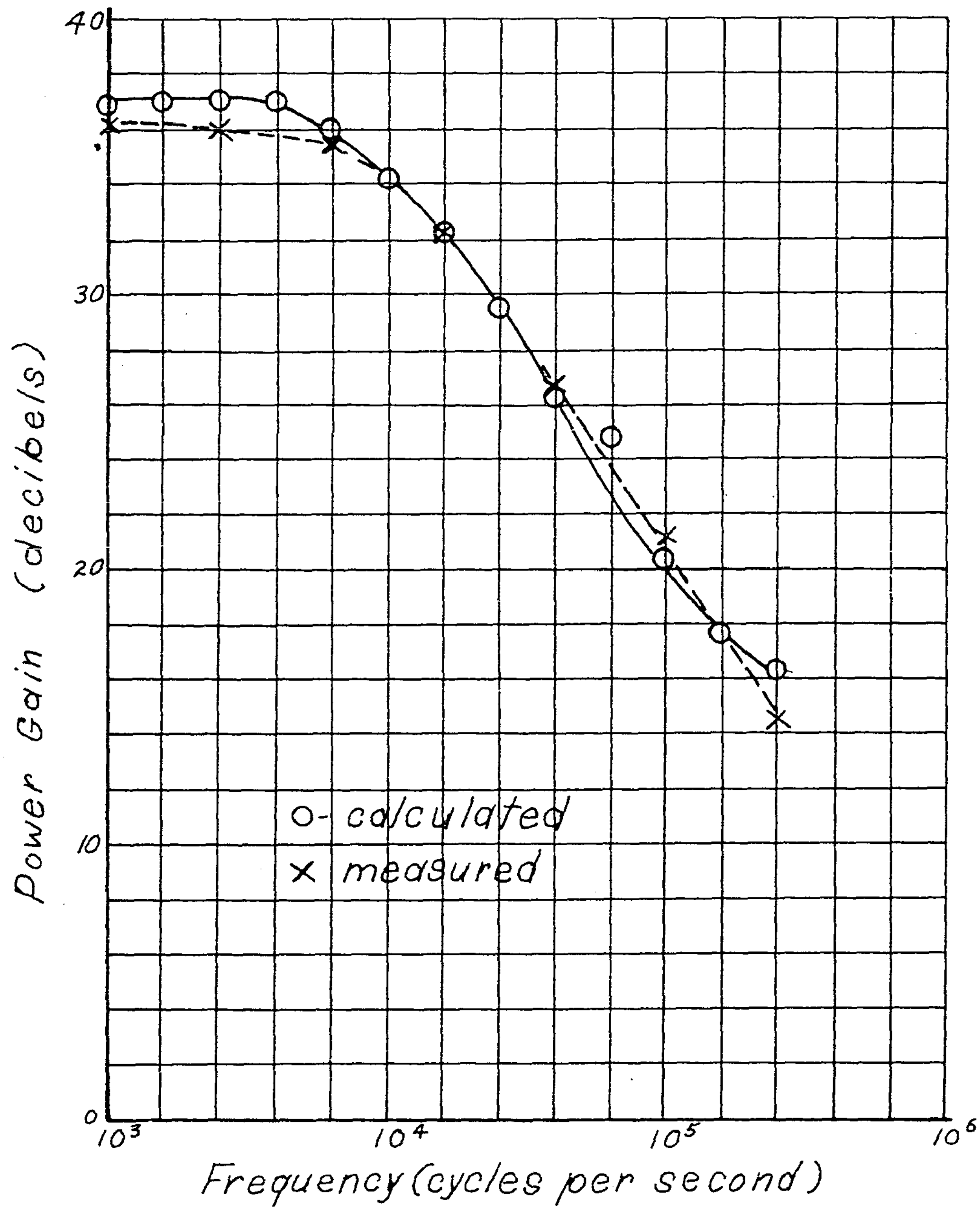


Figure 16. Power gain of neutralized transistor amplifier using Transistor 3

Table 3. Measured power gain for neutralized amplifier

Frequency cycles per second	Transistor 1 decibels	Transistor 3 decibels
1,000	34.6	36.18
2,510	34.4	36.03
6,310	33.99	35.62
15,850	31.38	32.16
39,800	26.28	26.62
100,000	20.77	21.28
251,000	15.14	14.58

3. The curves match very well from about 6,000 to 150,000 cycles per second.

C. Amplifier Operation with a Reflective Load Termination

In designing amplifiers, it is usually desirable to have the gain constant over a certain frequency range. For vacuum tube amplifiers, the ratio of the tube transconductance to the sum of the input and output capacitances of the tube gives an indication of the gain bandwidth product which can be attained when using the tube in an amplifier circuit. A simple gain bandwidth relationship does not exist for transistor amplifiers.

The power gain curves for conjugate-image operation are flat up to about ten kilocycles per second. If a larger bandwidth is desired, the transistor can be operated with a load admittance equal to that required for conjugate-image operation at a higher frequency. Reflections due to improper termination will reduce the gain at the lower frequencies and tend to give a more constant gain over a larger range of frequencies.

The load termination for conjugate-image operation at 100 kilocycles per second was used for the reflective load test. For Transistor 1 the load consisted of a 5,300 ohm resistor in series with 11.1 millihenries and for Transistor 3 the load was a 6,060 ohm resistor in series with 12.6 millihenries. The power gain was calculated using equation 54 and is tabulated in Table 4.

Table 4. Calculated power gain for a reflective termination

Frequency cycles per second	Transistor 1 decibels	Transistor 3 decibels
1,000	29.49	32.37
1,585	29.38	32.5
2,510	29.41	32.61
3,980	29.4	32.4
6,310	29.55	32.39
10,000	29.65	32.03
15,850	29.9	31.8
25,100	29.62	32.03
39,800	29.38	30.92
63,100	27.79	30.41
100,000	25.49	27.01
158,500	22.74	24.04
251,000	19.58	20.12
398,000	16.17	23.27

The schematic diagram of the circuit used to measure the power gain with a reflective termination is shown in Figure 17. The power output was determined in the same manner as was used previously, that is

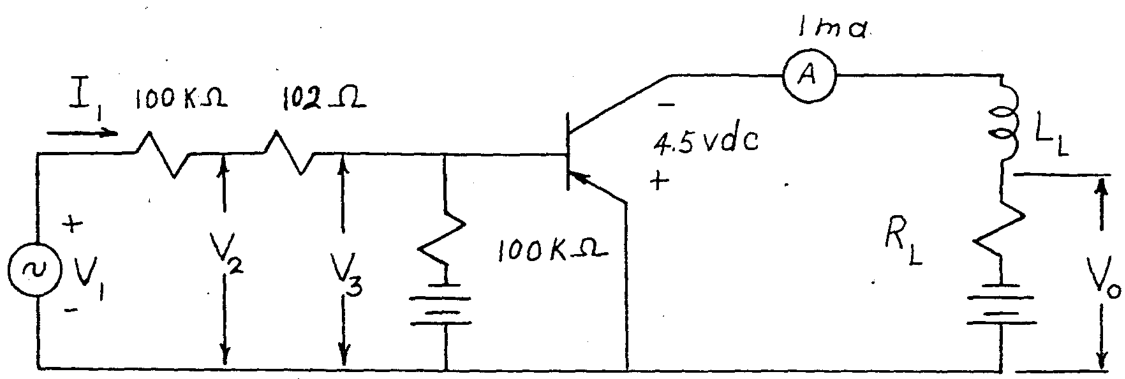


Figure 17. Schematic diagram of the circuit used to measure the power gain with a reflective load termination

Table 5. Measured power gain for reflective termination

Frequency cycles per second	Transistor 1 decibels	Transistor 3 decibels
1,000	30.02	32.9
1,585	30.02	32.69
2,510	30.15	32.56
3,980	30.08	32.56
6,310	30.00	32.44
10,000	30.05	32.32
15,850	29.93	32.07
25,100	29.76	31.50
39,800	28.89	30.91
63,100	27.81	29.23
100,000	25.7	26.95
158,500	22.88	23.57
251,000	16.58	16.95
398,000	10.45	6.76

$$P_o = \frac{V_o^2}{R_L}. \quad 140$$

The signal generator current, I_1, will be approximately V_1 divided by 10^5 ohms and the input power will be

$$P_{in} = V_3 I_1 \cos \theta. \quad 141$$

V_3 was measured and I_1 can easily be calculated, however the angle between the two is more difficult to calculate. The cosine of θ was determined using the cosine law as follows

$$\cos \theta = \frac{V_2^2 - V_3^2 - (102\ I_1)^2}{2V_3(102\ I_1)}. \quad 142$$

The power gains determined from the measurements are tabulated in Table 5.

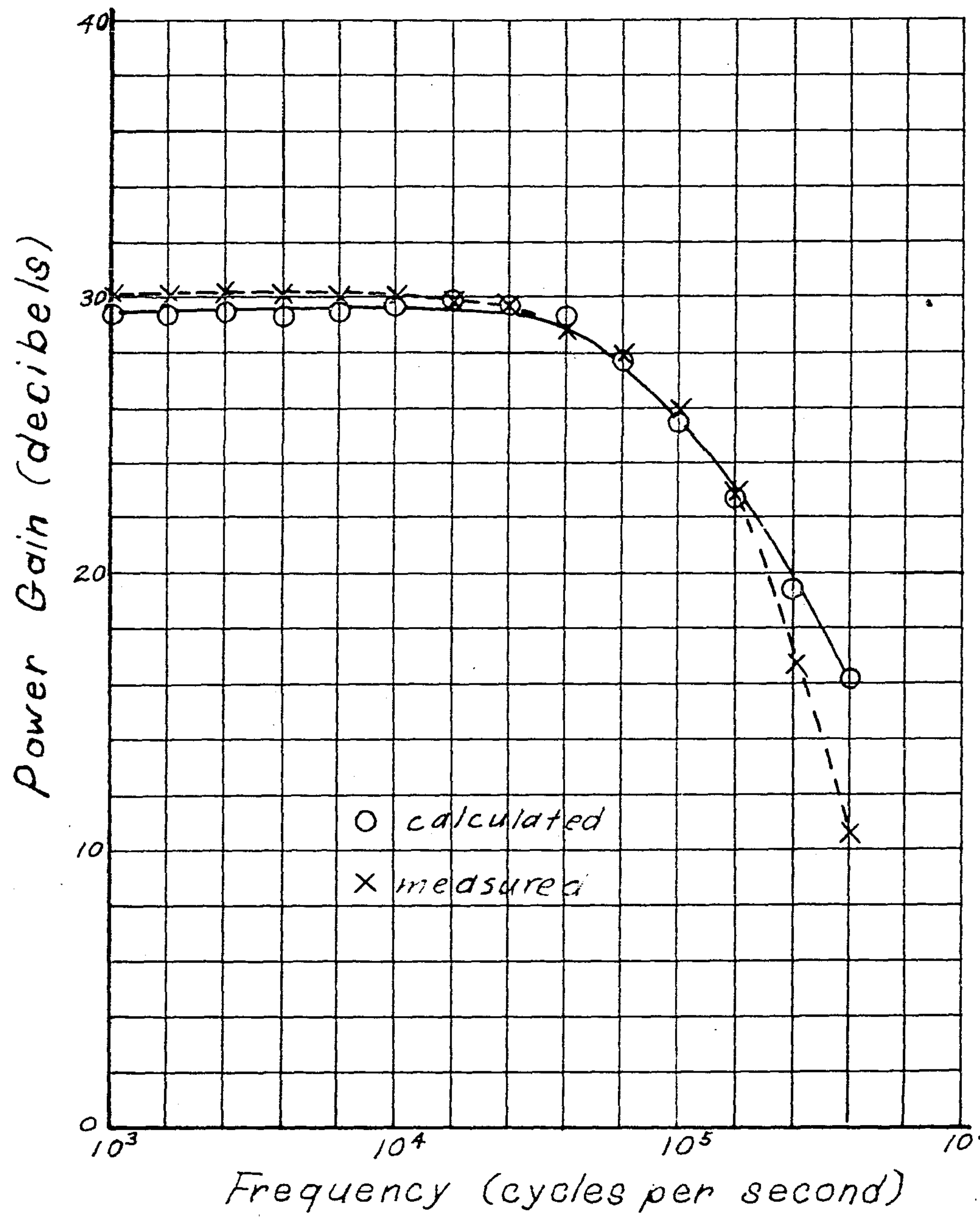


Figure 18. Power gain using Transistor 1 and a load impedance consisting of 5,300 ohms in series with 11.1 millihenries

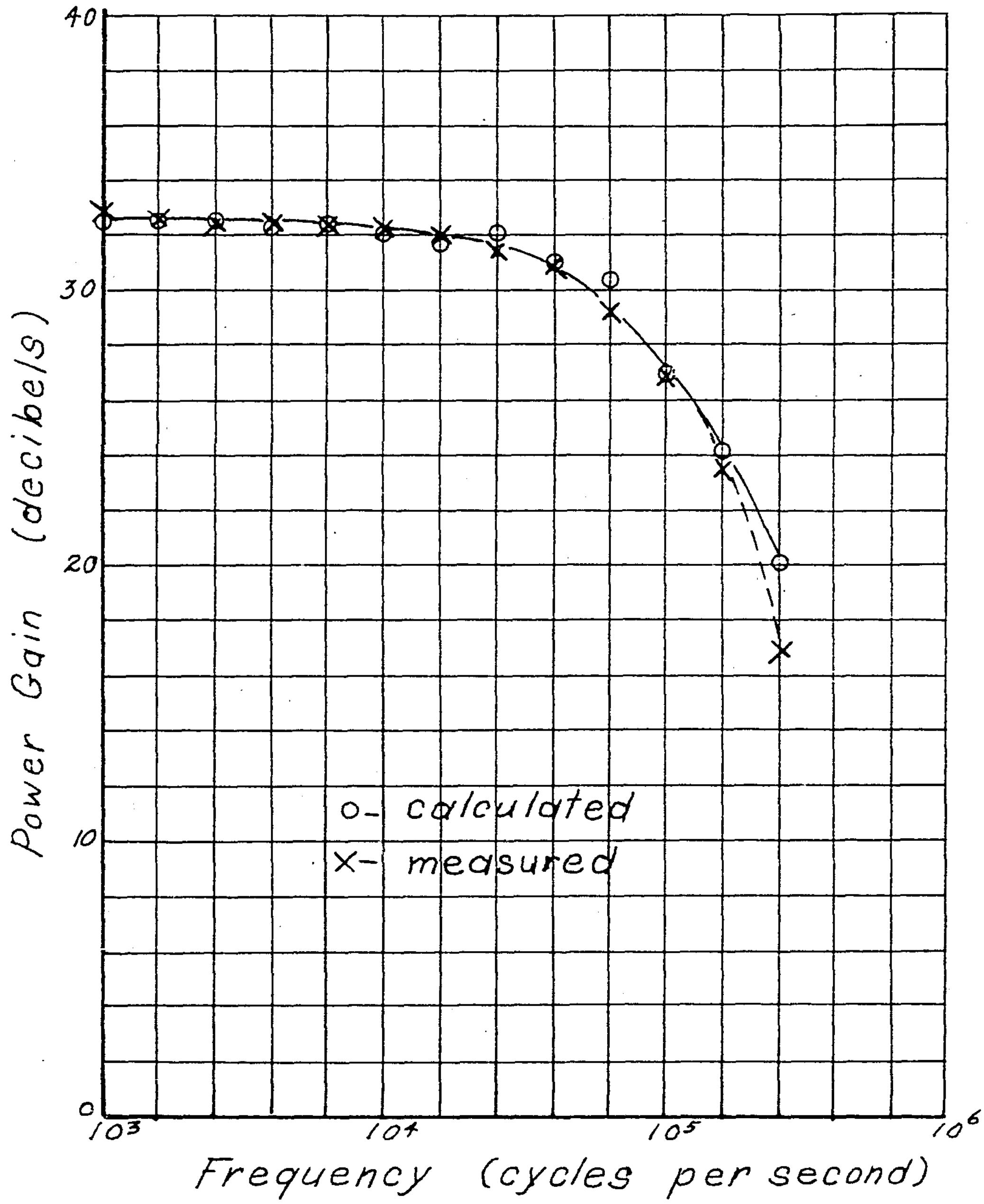


Figure 19. **Power gain using Transistor 3 and a load impedance consisting of 6,060 ohms in series with 12.6 millihenries**

The calculated and experimental power gains for Transistor 1 are plotted in Figure 18 and for Transistor 3 in Figure 19. The power gain levels off about five decibels above the 100 kilocycle per second power gain and six to seven decibels below the low frequency conjugate-image operation power gain. The three decibels bandwidth has increased from about 28,000 to about 75,000 cycles per second for Transistor 1 and from about 27,000 to about 60,000 cycles per second for Transistor 3. The gain bandwidth product for a reflective termination is less than for conjugate-image operation.

V. CASCADE OPERATION OF TRANSISTOR AMPLIFIERS

A. Impedance Matching for Conjugate-Image Operation

When the gain available from a single stage amplifier is not sufficient for a particular application, two or more stages are usually connected in cascade. Cascade operation involves connecting the output terminals of one stage of amplification to the input terminals of the following stage either directly or through an impedance matching network. The overall transmission matrix for networks in cascade is obtained by taking the matrix product of the individual transmission matrices of the component networks. The order for performing the matrix product is the same as the order of the networks in the cascade.

In determining the power gain of one network in a cascaded group of networks, the load impedance will be the input impedance of the following network. The power gain of the network will be the ratio of the power it supplies to the following network to the power it receives from the preceding network. Therefore, cascade of networks will have a total power gain equal to the product of the individual power gains.

The cascade operation of transistor amplifiers stages for maximum power transfer requires an impedance matching network. For the transistors measured, the input impedance was of the order of 300 ohms and the output impedance about 90,000 ohms at 1,000 cycles per second. Thus, the change in impedance level is rather large. If no impedance matching network were used, the reflection factor would be fairly close to one and the power gain considerably less than the matched power gain.

The design of an impedance matching network in the audio frequency range usually includes an audio transformer. O'Donnell and Williams (27) discuss impedance transformation using band-pass filters, however this method is not practical for the audio frequency range. Simple L-section or T-section matching networks designed for a single frequency will provide an approximate impedance match over a band of frequencies. However, the greater the change in impedance level the smaller the bandwidth. For an impedance level change from 300 to 90,000 ohms the bandwidth would be narrow, therefore an iron-cored audio transformer was used in the matching network between Transistor 3 and Transistor 1.

The low frequency response of an iron-cored transformer depends upon the amount of primary inductance relative to the primary impedance level. The higher the primary impedance level, the higher the primary inductance necessary to give the same low frequency response. The high frequency response will be influenced by the distributed capacitance of the transformer windings. The transformer used in the subsequent measurements was designed to provide a response flat within one half a decibel from 1,000 cycles per second to 25,000 cycles per second. The setting of a lower limit of 1,000 cycles per second was based on the preceding measurements. This was the lowest frequency for which the conjugate-image immittances and the power gain had been calculated. The primary winding of the transformer was divided into two coils, which were separated by the secondary winding. The division of the primary winding was to reduce the distributed capacitance and help achieve the desired bandwidth.

In section III an approximation was obtained for the load admittance of Transistor 3 for conjugate-image operation. This approximation was reasonably good up to about 100,000 cycles per second, however since the upper frequency limit was set at 25,000 cycles per second a simpler approximation can be used. For conjugate-image operation the output admittance will be the conjugate of the load admittance. The output admittance can be approximated by a 90,000 ohm resistor in parallel with a 300 micromicrofarad condenser.

The input admittance to Transistor 1 for conjugate-image operation will be Z_{c1}^*. It can be approximated by a negative inductance of 0.1065 millihenries in series with the parallel combination of a 242.5 ohm resistor and a 0.0314 microfarad condenser.

The impedance matching network was designed using the method of Fano (13) and Matthaei (26). First consider the output impedance of Transistor 3. The impedance and frequency scales are both normalized, the impedance level to 90,000 ohms and the frequency to 25,000 cycles per second. The normalized output impedance will then consist of

$$R_o = 1 \tag{143}$$

and

$$\begin{aligned} C_o &= 90{,}000 \times 2\pi \times 25{,}000 \times 300 \times 10^{-12} \\ &= 4.24. \end{aligned} \tag{144}$$

Then

$$A_1^{\infty} = \frac{2}{C_o} = 0.472 \tag{145}$$

which by using Figure 3 of Matthaei (26) indicates a

$$|\rho|_{max} = 0.55. \tag{146}$$

For this value of $|\rho|_{max}$ the loss in gain should not be more than about 1.5 decibels. The network obtained is shown in Figure 20. This network will give an impedance match such that $|\rho| < 0.55$ for frequencies up to

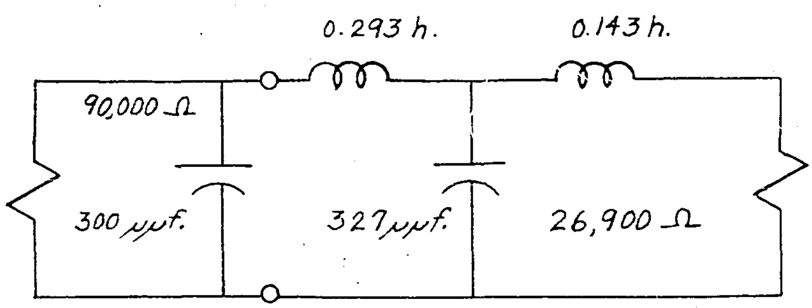


Figure 20. Matching network for output impedance of Transistor 3

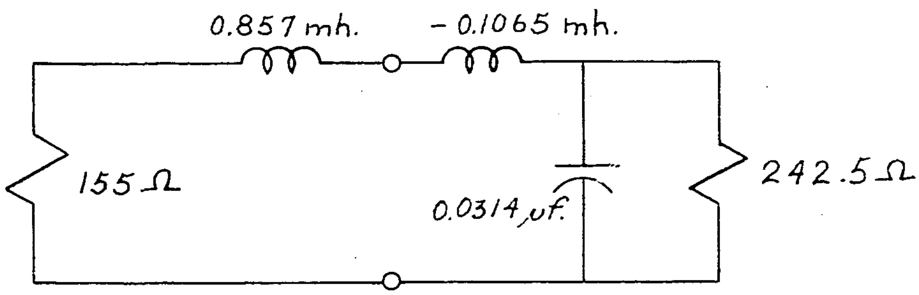


Figure 21. Matching network for input impedance of Transistor 1

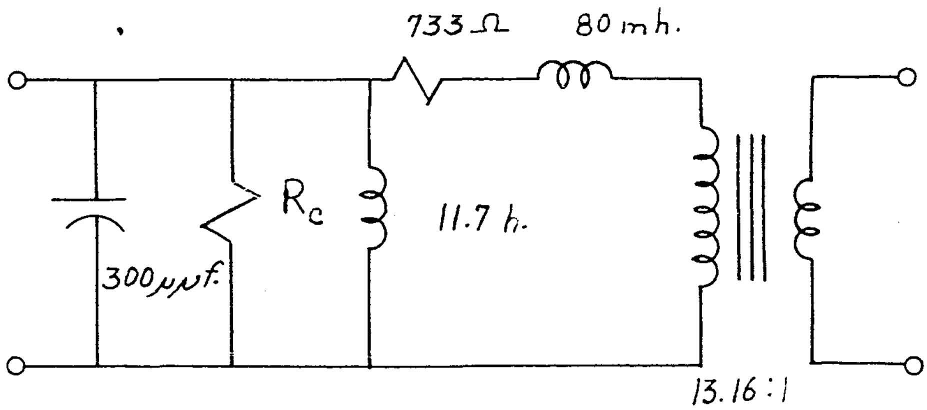


Figure 22. Equivalent circuit for the impedance matching transformer

25,000 cycles per second.

Now consider the input impedance to Transistor 1. Again it is necessary to normalize the impedance and frequency scales, however the impedance level is now normalized to 242.5 ohms. The normalized components of the input impedance will be

$$R_i = 1 \qquad 147$$

$$C_i = 2\pi\ 25{,}000 \times 90{,}000 \times 0.0314 \times 10^{-6}$$

$$= 1.198 \qquad 148$$

and

$$L_i = \frac{-0.1065 \times 10^{-3} \times 2\pi\ 25{,}000}{242.5}$$

$$= -0.0689. \qquad 149$$

Then

$$A_1^{\infty} = \frac{2}{1.198} = 1.67 \qquad 150$$

and for $n = 2$, $|\rho|_{max} = 0.223$. Even for a simpler matching network the reflection factor is considerably less than for the output impedance. The network obtained is shown in Figure 21.

Figures 20 and 21 indicate that the iron-cored audio transformer should be designed to operate from an impedance level of 26,900 ohms to 155 ohms. The iron core consisted of 16 mil EI laminations and was built up to form a 17/16 inch by 1 inch center leg. The primary consisted of 2184 turns of number 34 wire and the secondary a 166 turns of number 24 wire. An equivalent circuit for the transformer is shown in Figure 22. This equivalent circuit was based upon measurements made on the transformer.

The transformer leakage inductance and distributed shunt capacitance can be incorporated into the matching network. First reflect the 0.143 henry inductance of the output matching network into the secondary. Then the 300 micromicrofarad distributed capacitance of the transformer can be used for part of the 327 micromicrofarads needed in the matching networks. Also the 80 millihenries of leakage inductance can be reflected into the secondary and used as part of the reflected 0.143 henry inductance. A single inductance can be used for the 0.857 millihenry inductance needed in the input matching network and the remaining part of the reflected 0.143 henries. The complete matching network is shown in Figure 23. The power loss from this matching network should not be more than about 2.5 decibels. About 1.8 decibels due to reflections and 0.7 decibel due to losses in the transformer and inductors.

B. A Two Stage Transistor Amplifier

A two stage transistor amplifier was constructed using Transistor 3 in the first stage and Transistor 1 in the second stage. The impedance matching network discussed in the previous section was used to couple the output of Transistor 3 to the input of Transistor 1. A schematic diagram of the complete circuit is shown in Figure 24.

Both transistors stages were adjusted for a quiescent operating point of -4.5 volts for the collector voltage and -1 milliampere for the collector current. The output voltage was limited to two volts rms, so that operation would be over the linear portion of the transistor characteristics. Using a value of two volts for the output voltage and 90,000 ohms for the load resistance gives an output power of about 44 microwatts. The two stage amplifier should provide a power gain of about 74 decibels,

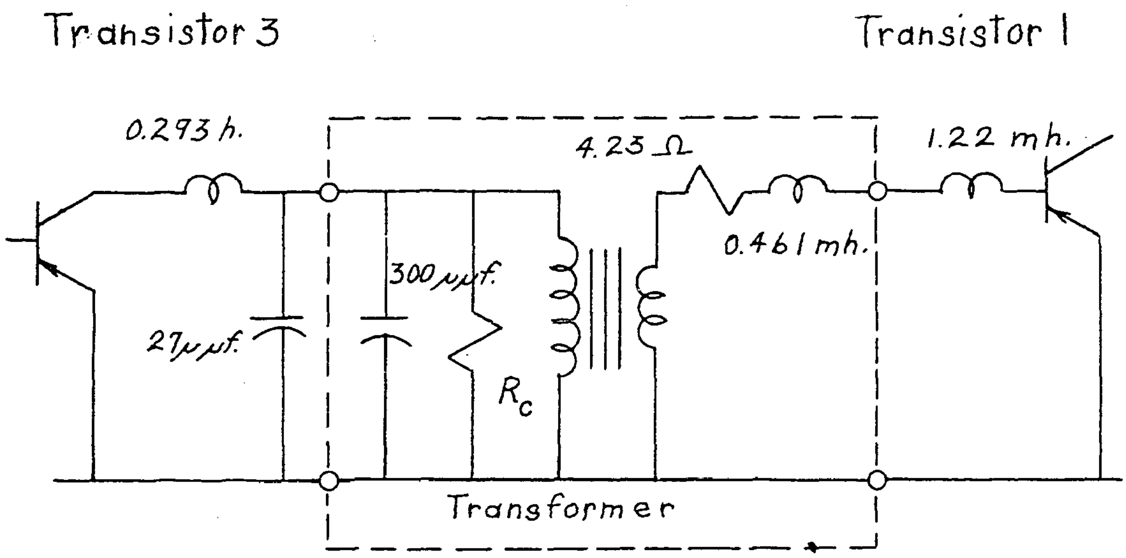


Figure 23. The impedance matching network

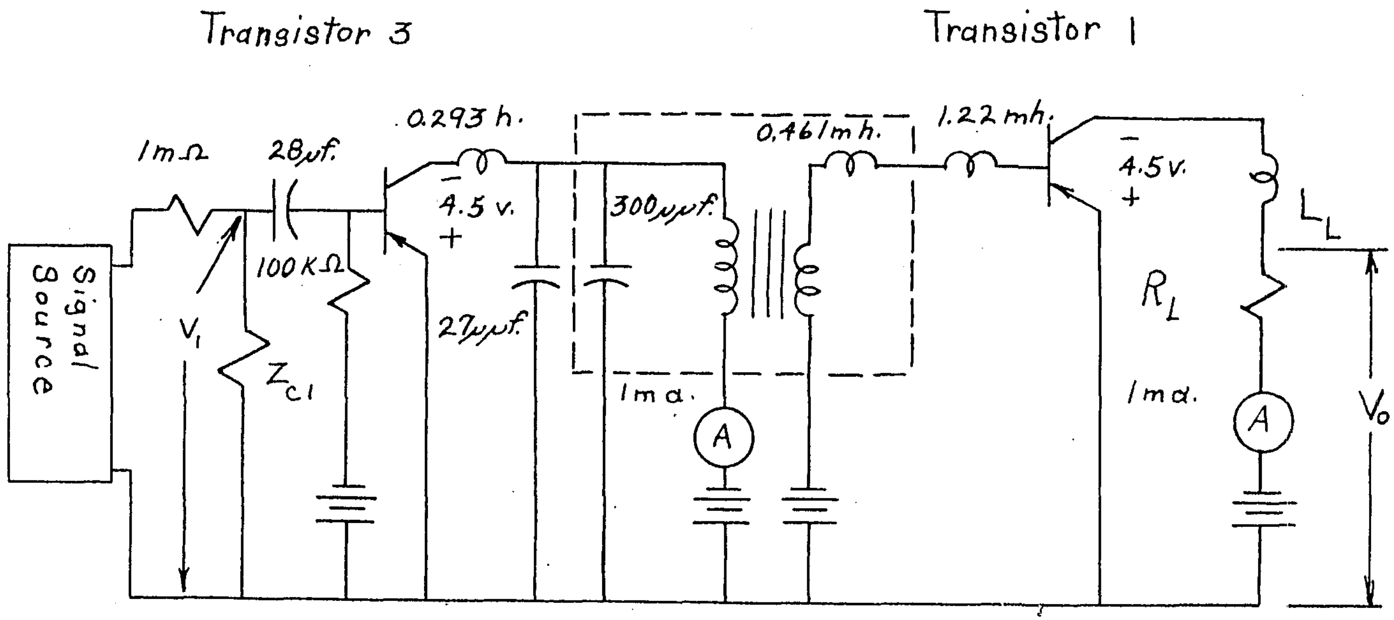


Figure 24. A two stage transistor amplifier

which would mean an input power of about 1.75 micromicrowatts or an input voltage of about 20 microvolts.

The noise voltage at the input is of the same order of magnitude as the signal voltage, which makes the measurement of the input voltage difficult. The amplifier section of a vacuum tube voltmeter was used to amplify the input voltage about 450 times before its measurement. A wave analyzer was used for the voltage measurement itself because its narrow bandwidth would filter out most of the noise and give an accurate reading for the signal voltage.

Table 6. Power gain for two stage transistor amplifier

Frequency cycles per second	Experimental decibels	Calculated decibels
1,000	73.5	74.8
1,585	73.88	74.82
2,510	74.04	74.94
3,980	73.81	74.76
6,310	72.78	74.46
10,000	70.83	74.58
15,850	72.76	72.74
25,100	69.75	70.19
39,800	57.57	64.81

The input power, output power and power gains were calculated from the measurements in the same manner that was used for single stage conjugate-image operated amplifiers. The experimental and calculated power gains are tabulated in Table 6.and plotted in Figure 25.

The difference between the experimental and the calculated power

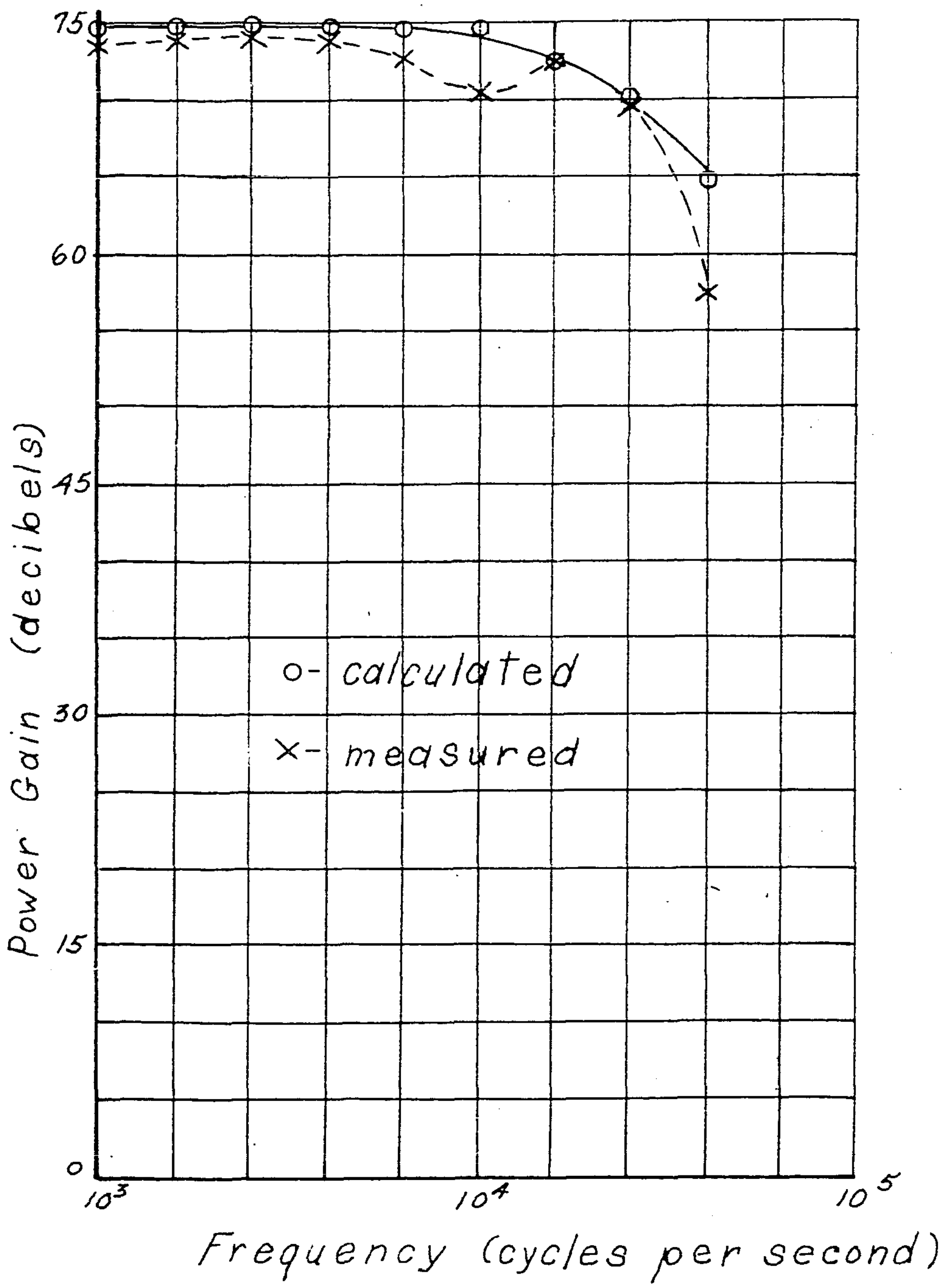


Figure 25. Power gain for a two stage transistor amplifier

gains should be about 2.5 decibels at the points of maximum reflection factor. The larger deviation in power gain at 10,000 cycles per second indicates the maximum reflection coefficient was probably larger than the predicted 0.55. Another source of error would be caused by Transistor 3 not being terminated in its conjugate-image impedance and, therefore, its input impedance will not be Z_{c1}^*. Likewise, since the source impedance for Transistor 1 is not exactly Z_{c1} its output admittance will not be exactly Y_{c2}^*. The small signal-to-noise ratio at the input could be the source of some error in measuring the input voltage. Also, the low signal-to-noise ratio at the output, while larger than that at the input, could cause some error in measuring the output voltage.

The two stage transistor amplifier provided a maximum of about 74 decibels gain and a bandwidth of about 25,000 cycles per second. The experimental response curve has the proper shape for a Tchebyscheff response of order n = 4.

VI. SUMMARY

The conjugate-image operation of a two terminal pair network maximizes the available power gain. For such operation the source impedance is the conjugate of the input impedance and the load impedance is the conjugate of the output impedance. The expressions for the source impedance and load admittance for conjugate-image operation are developed in terms of the small signal h-parameters.

The operation of a two terminal pair network for maximum available power gain is a very desirable type of operation for communication networks, where usually the amount of the signal power transferred is more important than the efficiency of the transfer. The power gain that was used in this thesis was the ratio of the output power to the input power, which will be the same as the available power gain when the source and load terminations are those for conjugate-image operation.

Operation for maximum available power gain presents some problems, which limit the frequency range of operation. Since the source impedance is the conjugate of the input impedance, one or both of these impedances may not be physically realizable over the complete range of frequencies. However, they can be approximated over some finite frequency range. The narrower the frequency range the better the approximation. The same situation exists relative to the load admittance and the output admittance.

The conjugate-image immittances and the maximum available power gain provide useful information for designing transistor amplifiers. For example, if a three decibel bandwidth were needed that was larger than that given by conjugate-image operation, the conjugate-image data would give an indication of the power gain available and the source and load terminations needed for that bandwidth. Reflections caused by improper termination for conjugate-image operation will cause the power gain to remain fairly constant over the bandwidth. For three decibel bandwidth the conjugate-image load termination would have to be that for higher frequency than indicated by the bandwidth. The experimental and calculated results for operation with reflection indicated a rise of about four or five decibels above the 100,000 cycle per second power gain.

In general, junction transistors will not be stable for conjugate-image operation at all frequencies. However, transistors can be neutralized and made stable at all frequencies. The neutralizing network will generally affect all of the h-parameters and not h_{12} alone. Thus, the neutralized amplifier power gain depends to some extent on the neutralizing network.

The cascade operation of transistor amplifiers for maximum available power gain with a certain reflection factor is possible. The reflection factor depends upon the bandwidth desired and the input and output impedances to be matched. A 74 decibel power gain was obtained from a two stage transistor amplifier with a 25,000 cycle per second upper frequency limit.

VII. ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor W. L. Cassell and Professor W. B. Boast for their excellent suggestions and criticisms.

VIII. LITERATURE CITED

1. Armstrong, H. L. An approximate treatment of cascaded four-terminal networks. Electronic Engineering. 27: 130-131. 1955.

2. _______. Note on the use of Tchebycheff functions in dealing with iterated networks. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-2: 169-170. 1955.

3. Bardeen, J. and Brattain, W. H. Transistor, a semiconductor triode. Phys. Rev. 74: 230-231. 1948.

4. Bode, Hendrik W. Network analysis and feedback amplifier design. 1st ed. New York, D. Van Nostrand Co., Inc. 1945.

5. Brown, J. S. and Bennett, F. D. The application of matrices to vacuum tube circuits. Proc. of the Inst. Radio Engrs. 36: 844-852. 1948.

6. Bruun, Georg. Common-emitter transistor video amplifiers. Proc. of the Inst. Radio Engrs. 44: 1561-1572. 1956.

7. Carlin, H. J. and La Rosa, R. Broadband reflectionless matching with minimum insertion loss. In Proc. of the Symposium on Modern Network Synthesis. pp. 161-178. New York, Polytechnic Institute of Brooklyn. 1952.

8. Cheng, C. C. Neutralization and unilateralization. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-2: 138-145. 1955.

9. Chu, Ge Yao. A new equivalent circuit for junction transistors. Convention record of the Inst. Radio Engrs. Vol. 2, Part 2: 135-140. 1954.

10. _______. Unilateralization of junction-transistor amplifiers at high frequencies. Proc. of the Inst. Radio Engrs. 43: 1001-1006. 1955.

11. Coffey, W. N. Measuring r-f parameters of junction transistors. Electronics. 29: 152-155. Feb. 1956.

12. Drouilhet, P. R. Predictions based on the maximum oscillator frequency of a transistor. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-2: 178-183. 1955.

13. Fano, R. M. Theoretical limitations on the broadband matching of arbitrary impedances. Jour. of Franklin Inst. 249: 57-83, 139-154. 1950.

14. Fougere, G. L. Maximum power transfer in transistor amplifiers. Electronic Engineering. 27: 492-494. 1955.

15. Gade, Donald W. Feedback and stability of junction transistor circuits. Iowa State College. Engr. Exp. Sta. Engineering Report No. 19. 1953-1954.

16. Giacoletto, L. J. Equipment for measuring junction transistor admittance parameters for a wide range of frequencies. RCA Review. 14: 269-296. 1953.

17. ________. Study of pnp alloy junction transistors from d.c. through medium frequencies. RCA Review. 15: 506-563. 1954.

18. Guillemin, Ernst A. Communication networks, Vol. 2. 1st ed. New York, John Wiley and Sons, Inc. 1935.

19. ________. A summary of modern methods of network synthesis. In Marton, L., ed. Advances in electronics, Vol. 3. pp. 261-303. New York, Academic Press, Inc. 1951.

20. Hunter, L. P. Handbook of semiconductor electronics. 1st ed. New York, McGraw-Hill Book Co., Inc. 1956.

21. Knausenberger, G. E. A note on the scattering matrix of an active linear two-terminal pair network. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-2: 112-116. 1955.

22. Knight, Geoffrey, Johnson, R. A. and Holt, R. B. Measurement of the small signal parameters of transistors. Proc. of the Inst. Radio Engrs. 41: 983-989. 1953.

23. La Rosa, R. and Carlin, H. J. A general theory of wideband matching with dissipative 4 poles. Jour. of Math. and Phys. 33: 331-345. 1955.

24. Linvill, J. G. and Schimpf, L. G. The design of tetrode transistor amplifiers. Bell Sys. Tech. Jour. 35: 813-840. 1956.

25. Lo, A. W., Endres, R. O., Zawels, Jakob, Waldhauer, F. D. and Cheng, C. C. Transistor Electronics. 1st ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1955.

26. Matthaei, G. L. Synthesis of Tchebycheff impedance-matching networks, filters and interstages. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-3: 163-172. 1956.

27. O'Donnell, T. J. and Williams, E. M. Maximum impedance transformations in band-pass filters. Communications and Electronics, Amer. Inst. of Elect. Engr. No. 6: 110-113. May 1953.

28. Pipes, Louis A. The matrix theory of four-terminal networks. Lond., Edin. and Dublin Phil. Mag. Ser. 7. 30: 370-395. 1940.

29. Pritchard, R. L. Frequency response of theoretical models of junction transistors. Transactions on Circuit Theory, Prof. Gr. Cir. Th. of Inst. Radio Engrs. CT-2: 183-191. 1955.

30. ________. High-frequency power gain of junction transistors. Proc. of the Inst. Radio Engrs. 43: 1075-1085. 1955.

31. ________. Measurement considerations in high-frequency power gain of junction transistors. Proc. of the Inst. Radio Engrs. 44: 1050-1051. 1956.

32. Roberts, Shepard. Conjugate-image impedances. Proc. of the Inst. Radio Engrs. 34: 198-204. 1946.

33. Shea, R. F. Transistor circuit engineering. 1st ed. New York, John Wiley and Sons, Inc. 1957.

34. Shekel, Jacob. Matrix representation of transistor circuits. Proc. of the Inst. Radio Engrs. 40: 1493-1497. 1952.

35. ________. Reciprocity relations in active 3-terminal elements. Proc. of the Inst. Radio Engrs. 42: 1268-1270. 1954.

36. Statz, H., Guillemin, E. A. and Pucel, R. A. Design considerations of junction transistors at higher frequencies. Proc. of the Inst. Radio Engrs. 42: 1620-1628. 1954.

37. Stern, Arthur P. Stability and power gain of tuned transistor amplifiers. Proc. of the Inst. Radio Engrs. 45: 335-343. 1957.

38. ________, Aldridge, C. A. and Chow, W. F. Internal feedback and neutralization of transistor amplifiers. Proc. of the Inst. Radio Engrs. 43: 838-847. 1955.

39. Storch, Leo. The transmission matrix of n alike cascaded networks. Communication and Electronics, Amer. Inst. of Elect. Engr. No. 16: 616-619. Jan. 1955.

40. Wallace, R. L., Jr. and Peitenpol, W. J. Some circuit properties and applications of npn transistors. Proc. of the Inst. Radio Engrs. 39: 753-767. 1951.

41. Zawels, Jakob. Physical theory of new circuit representation for junction transistors. Jour. of Applied Physics. 25: 976-981. 1954.

IX. APPENDICES

A. Measurement of the Small Signal h-parameters

Several articles have been written discussing the measurement of small signal junction transistor parameters (11)(16)(22). Some of the articles discuss commercial pieces of equipment, while others are of a more general nature. Giacoletto (16) considers the measurement of the small signal admittance parameters for frequencies up to one megacycle. The methods used for the measurements discussed in this appendix involve the adaption of Giacoletto's methods to measure the h-parameters.

Static output characteristics and input characteristics were run on four ZJ3-1 transistors for common-emitter operation. From a study of the plotted characteristics an operating point of $V_c = -4.5$ volts and $I_c = -1$ milliampere was chosen.

The common-emitter h_{11e} and h_{22e} were measured using bridge techniques. The output of the bridge was fed to a difference amplifier which in turn was coupled to a pentode amplifier. The total detection amplifier had a gain of about 45 decibels. The circuits used for measuring h_{11e} and h_{22e} are shown in Figures 26 and 27 respectively and the data obtained are tabulated in Tables 7 and 8 respectively.

In Figure 26 the impedance from base to emitter will be equal to h_{11e} if the signal voltage from collector to emitter is zero. The impedance and thus the signal voltage from collector to emitter was kept low by using a 4.5 volt battery as the bias source and then shunting it with a large condenser, so that

$$h_{11e} = \frac{1}{1/R + j\omega C}. \qquad 151$$

In Figure 27 a one megohm resistor is placed in the base lead to make the

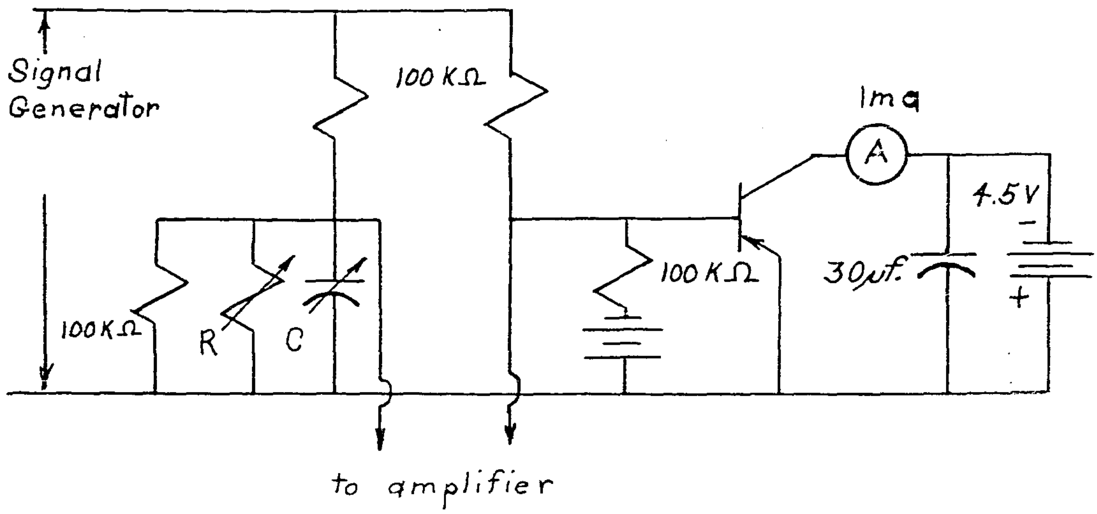


Figure 26. Circuit for measuring h_{11e}

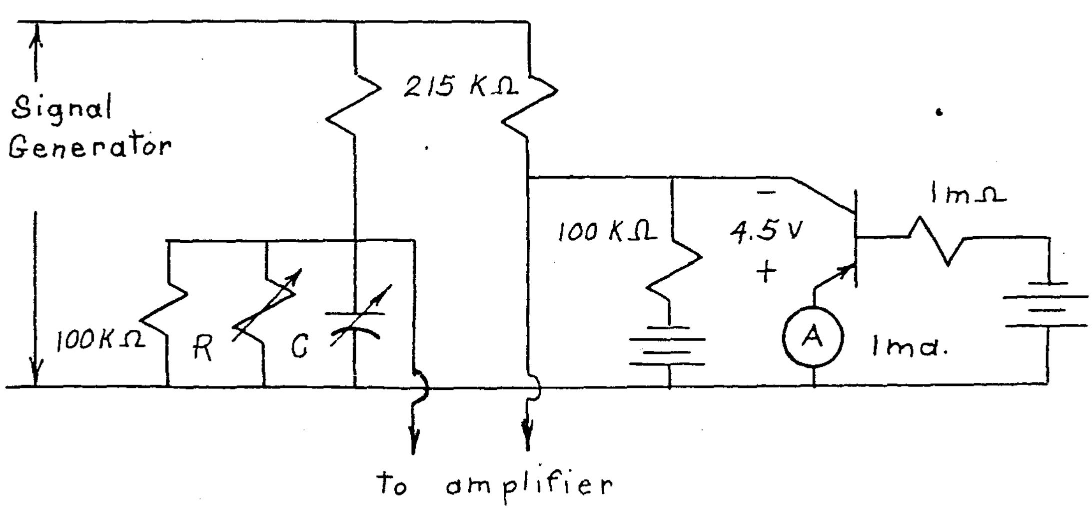


Figure 27. Circuit for measuring h_{22e}

Table 7. h_{11e} in ohms

Frequency cycles per second	Transistor 1	Transistor 2	Transistor 3	Transistor 4
1,000	270.0/0°	700.0/0°	365.0/-1.58°	400.0/-1.44°
1,585	270.0/0°	700.0/-2°	360.0/-2.46°	390.0/-2.22°
2,510	270.0/-1.47°	700.0/-3.17°	360.0/-3.91°	380.0/-3.44°
3,980	270.0/-3.87°	685.0/-6.9°	358.0/-6.16°	380.0/-5.45°
6,310	271.0/-5.7°	670.0/-10.5°	355.0/-9.72°	377.0/-8.16°
10,000	270.0/-8.3°	636.0/-15.31°	344.5/-15.1°	372.4/-11.72°
15,850	258.5/-12.6°	587.0/-23.8°	321.8/-21.66°	353.5/-17.8°
25,100	238.0/-17.53°	509.0/-32.65°	280.0/-30.56°	320.0/-25.76°
39,800	219.2/-24.24°	403.0/-41.76°	224.2/-38.1°	267.5/-33.86°
63,100	182.2/-29.88°	304.0/-49.2°	170.0/-42.3°	210.0/-39.8°
100,000	139.0/-29.82°	219.6/-51.8°	123.9/-41.36°	157.0/-42°
158,500	115.2/-27.4°	153.0/-48.3°	97.5/-35.63°	121.0/-38.8°
251,000	102.0/-21.8°	117.0/-43.06°	74.9/-28.3°	98.1/-32.9°
398,000	77.8/-20.5°	80.0/-36.85°	60.0/-24.8°	70.3/-27.2°
631,000	60.0/-20.88°	77.5/-35.6°	49.5/-20.7°	55.7/-22.05°
1,000,000	47.8/-23°	76.5/-31.96°	43.9/-17.65°	48.5/-21.44°

Table 8. h_{22e} in micromhos

Frequency cycles per second	Transistor 1	Transistor 2	Transistor 3	Transistor 4
1,000	12.47/7.6°	17.6/15.68°	13.92/10.42°	10.84/13.98°
1,585	12.62/11.92°	19.0/22.9°	14.27/16.28°	11.85/20.52°
2,510	13.0/18.47°	21.6/32.9°	15.08/24.85°	12.25/30.7°
3,980	14.43/25.8°	26.75/44.1°	17.78/34.4°	14.0/42.1°
6,310	17.9/40.3°	37.5/36.8°	23.8/44.5°	21.84/52°
10,000	24.5/49.3°	52.3/58.6°	34.1/51.55°	33.5/55.45°
15,850	35.6/55°	76.8/57.9°	48.5/51.6°	52.6/58.75°
25,100	52.0/55°	109.0/52.9°	65.9/46.6°	76.0/54.8°
39,800	76.5/50.4°	144.0/43.8°	83.0/38.4°	102.0/47.15°
63,100	99.4/45.4°	177.4/37.5°	96.4/33.24°	127.0/40.08°
100,000	121.4/39.6°	204.0/28.8°	107.2/30.6°	146.2/33.4°
158,000	138.0/34.4°	215.0/26.7°	108.2/28.6°	153.8/28.64°
251,500	121.8/33.9°	211.6/18.72°	115.0/31.5°	150.0/24.22°
398,000	142.0/44.75°	216.5/27.5°	135.2/41.74°	163.8/34.4°
631,000	173.2/55.2°	239.5/36.6°	176.5/53.8°	200.4/45.4°
1,000,000	246.5/66.35°	296.0/49.6°	249.0/65.25°	266.0/58.06°

the signal current in that lead approximately zero. Then the admittance from collector to emitter will be h_{22e} in parallel with the 100 kilohm resistor used in supplying DC operating voltage. The effect of the 100 kilohm resistor is balanced by the 100 kilohm resistor in parallel with the variable resistance and capacitance used to balance the bridge. Then

$$h_{22e} = 1/R + j\omega C. \tag{152}$$

The schematic diagram and the equivalent circuit for measuring h_{21e} are shown in Figure 28. Assuming $R_1 \gg h_{11}$, the following equations can be written

$$I_1 = V_1(G_1 + G_2 + Y_x) - V_c(Y_x + h_{12e}G_1) \tag{153}$$

$$0 = h_{21e}I_b - V_1Y_x + V_c(h_{22e} + g_L + Y_x) \tag{154}$$

$$I_b = (V_1 - h_{12e}V_c)G_1 \tag{155}$$

substituting 155 into 154 and collecting terms one has

$$0 = (h_{21e}G_1 - Y_x)V_1 + V_c(h_{22e} + g_L + Y_x - h_{21e}h_{12e}G_1). \tag{156}$$

Adjusting Y_x to make $V_c = 0$, then

$$h_{21e} = \frac{Y_x}{G_1}. \tag{157}$$

The results of these measurements are contained in Table 9.

The most difficult parameter to measure was h_{12e}, because of its small value. The circuits involved in the measurement of h_{12e} are shown in Figure 29. A phase splitting network was constructed to supply the balanced input and measurement indicated good balance over the frequency range used. Summing the currents at the V_2 node gives the following equation

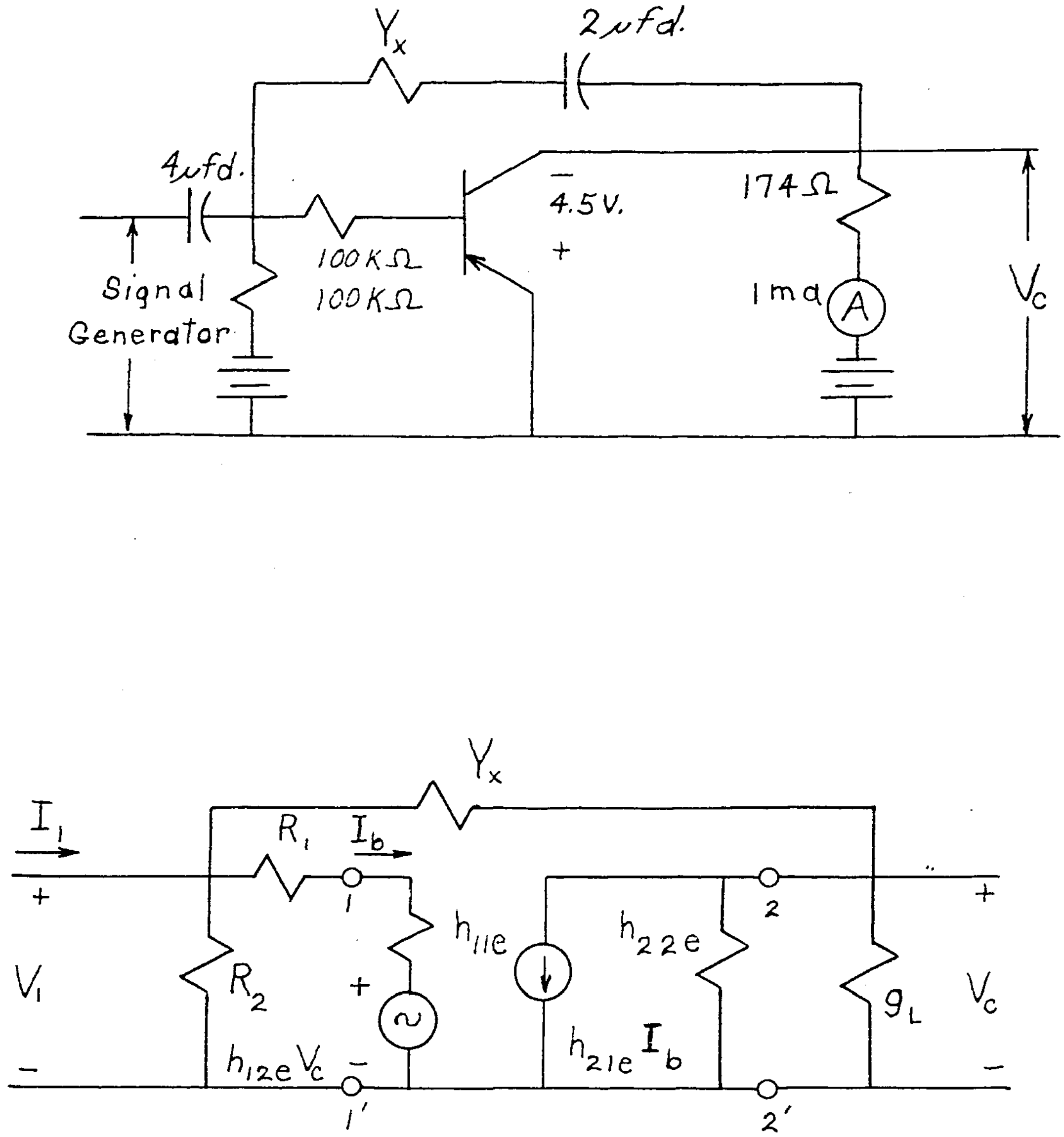


Figure 28. Circuit for measuring h_{21e}

Table 9. h_{21e}

Frequency cycles per second	Transistor 1	Transistor 2	Transistor 3	Transistor 4
1,000	7.14/-1.26°	21.45/-2.94°	10.88/-2.49°	11.53/-1.95°
1,585	7.04/-2.03°	21.45/-3.6°	10.9/-3.41°	11.47/-2.86°
2,510	7.04/-3.22°	21.2/-5.04°	10.9/-5.0°	11.44/-3.96°
3,980	7.04/-5.1°	21.18/-7.54°	10.8/-7.84°	11.4/-6.24°
6,310	7.02/-7.95°	20.43/-11.03°	10.62/-12.34°	11.26/-9.83°
10,000	7.0/-12.13°	19.85/-17.6°	10.29/-19.1°	11.0/-15.4°
15,850	6.74/-19.2°	18.6/-26.7°	9.62/-28.6°	10.52/-23.5°
25,100	6.35/-28.6°	16.34/-38°	8.45/-41.4°	9.49/-34.4°
39,800	5.7/-40°	13.2/-50.8°	6.67/-54.6°	8.0/-46.8°
63,100	4.66/-55.4°	9.93/-62.2°	5.9/-62.4°	6.25/-60.1°
100,000	3.5/-69.5°	7.03/-71.45°	3.5/-77.14°	4.7/-72.05°
158,500	2.74/-79.22°	4.93/-77.75°	2.6/-83.3°	3.32/-80.95°
251,000	2.36/-84.3°	3.7/-81.65°	2.12/-85.5°	2.64/-83.96°
398,000	2.39/-86.4°	3.33/-83.84°	2.38/-86.8°	2.75/-85.9°
631,000	3.18/-86.9°	3.41/-84.4°	3.52/-87.4°	3.48/-87.1°
1,000,000	7.16/-86.3°	4.6/-85.1°	9.83/-86.16°	6.37/-86.35°

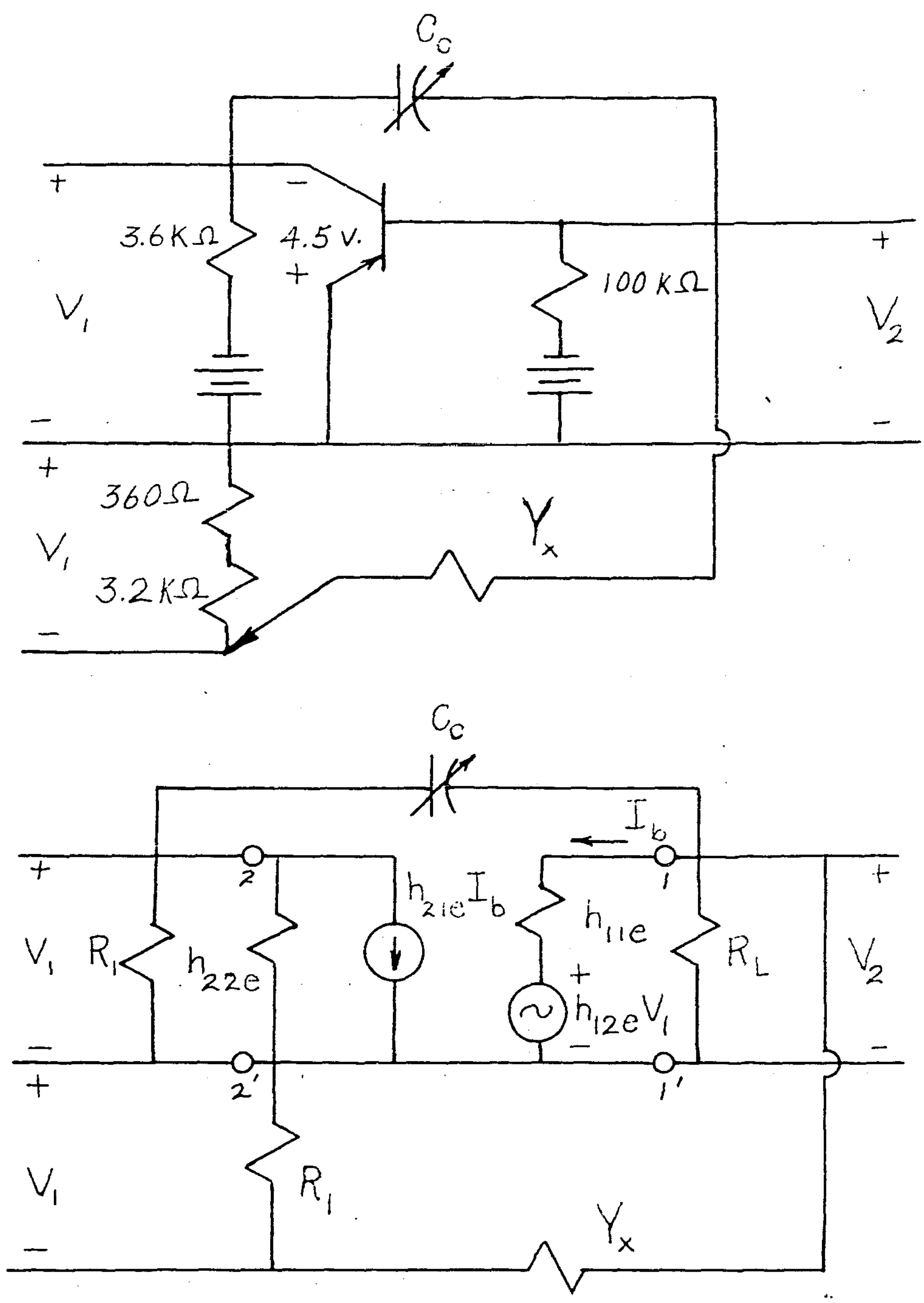


Figure 29. Circuit for measuring h_{12e}

Table 10. $h_{12e} \times 10^4$

Frequency cycles per second	Transistor 1	Transistor 2	Transistor 3	Transistor 4
1,000	0.934/15.55°	2.12/34.2°	1.6/22.6°	1.63/34°
1,585	1.078/33.4°	2.82/49.7°	1.7/29.5°	1.6/33.6°
2,510	1.215/40.8°	3.97/60.5°	1.99/39.7°	1.94/45.8°
3,980	2.15/61.4°	5.24/64°	3.02/55.5°	3.08/60.4°
6,310	3.35/65.5°	7.42/66.4°	4.4/61.5°	4.6/66°
10,000	4.09/66.4°	10.54/64.7°	6.63/59.8°	6.6/65.2°
15,850	5.98/64.9°	14.68/58.5°	9.01/58°	9.64/61.6°
25,100	8.75/59.4°	20.15/51°	11.98/45.9°	13.7/54.9°
39,800	12.0/50.6°	25.1/40.5°	15.75/38.1°	17.75/47.4°
63,100	14.6/41.9°	29.4/32.3°	18.2/32.2°	21.9/36.3°
100,000	16.5/35.2°	32.55/27.4°	17.9/29.4°	24.4/31.8°
158,500	18.7/38.2°	34.2/27.32°	19.7/30°	27.2/31.2°
251,000	18.7/41.1°	34.04/26.4°	16.95/35.5°	26.9/30°
398,000	16.3/46°	29.36/36.7°	15.6/42.6°	21.55/38.7°
631,000	14.52/48.9°	29.0/38.9°	15.15/50.2°	18.6/46°
1,000,000	14.08/50.5°	33.4/44.8°	18.4/58.6°	20.9/51.8°

$$0 = (Y_x - \frac{h_{12e}}{h_{11e}} - j\omega C_c)V_1 + (\frac{1}{h_{11e}} + g_L + Y_x + j\omega C_c)V_2. \qquad 158$$

Y_x is adjusted to make $V_2 = 0$, then

$$h_{12e} = h_{11e}(Y_x - j\omega C_c). \qquad 159$$

The values determined for h_{12e} are contained in Table 10.

B. Tabulation of Calculated Conjugate-Image Immittances

Table 11. Z_{c1} in ohms

Frequency cycles per second	Transistor 1	Transistor 3	Transistor 4
1,000	242.5/1.56°	301.0/6°	316.6/10.48°
1,585	242.0/3.8°	295.8/8.8°	308.4/11.24°
2,510	242.2/6.61°	292.2/13.8°	297.5/18.28°
3,980	240.0/16.04°	285.0/25.1°	285.2/37°
6,310	230.0/25°	275.0/36.5°	282.0/49.3°
10,000	225.2/29.7°	237.0/55.2°	266.0/56.6°
15,850	200.0/39.6°	212.5/63°	227.5/77.9°
25,100	164.8/46.6°	165.0/66.2°	191.0/85.76°
39,800	150.5/43.2°	129.0/63.6°	162.8/67.25°
63,100	128.2/38.4°	82.6/78°	125.0/57.85°
100,000	103.7/29.95°	83.5/42.7°	99.8/48.55°
158,500	88.1/24.7°	67.0/31.7°	81.9/38°
251,000	79.4/16.8°	55.1/22.8°	70.0/27.55°
398,000	57.0/14.9°	39.0/18.4°	45.0/20.95°
631,000	30.1/11.3°	6.4/18.3°	25.6/13.16°

Table 12. Y_{c2} in micromhos

Frequency cycles per second	Transistor 1	Transistor 3	Transistor 4
1,000	11.18/-6.91°	11.36/-8.71°	8.33/-9.5°
1,585	11.23/-9.63°	11.5/-14.45°	8.7/-19.8°
2,510	11.44/-16.04°	11.79/-23.15°	8.91/-28.8°
3,980	11.8/-19.6°	12.52/-31.4°	7.82/-47.5°
6,310	13.4/-36.2°	15.85/-47.3°	13.22/-60°
10,000	18.42/-50.6°	21.8/-66.7°	21.4/-69°
15,850	26.4/-61.9°	32.4/-72.55°	35.9/-83.8°
25,100	39.16/-67.8°	47.9/-74.94°	54.3/-87.7°
39,800	61.0/-63.9°	63.5/-70.6°	77.0/-75.2°
63,100	87.3/-58.95°	75.0/-81.6°	106.7/-68°
100,000	114.4/-52.6°	100.4/-52.5°	131.6/-58.4°
158,500	124.5/-49.3°	103.6/-48.6°	143.8/-50.1°
251,000	116.4/-46°	110.9/-47°	141.0/-43.2°
398,000	140.0/-57°	132.6/-58.9°	156.2/-54.5°
631,000	176.2/-72.8°	177.6/-85.56°	196.1/-69.2°

C. Calculated Power Gain

The measured h-parameters tabulated in Appendix A and the calculated conjugate-image immittances tabulated in Appendix B were used in equation 30 to calculate power gain. These results are contained in Table 13.

Table 13. Calculated power gain in decibels for conjugate-image operation

Frequency cycles per second	Transistor 1	Transistor 3	Transistor 4
1,000	36.27	38.53	39.98
1,585	36.16	38.66	40.06
2,510	36.18	38.76	40.3
3,980	36.1	38.66	40.83
6,310	36.06	38.4	40.21
10,000	35.77	38.81	39.13
15,850	35.2	37.54	39.3
25,100	34.16	36.03	37.94
39,800	31.41	33.4	34.03
63,100	28.6	34.01	30.9
100,000	25.49	27.01	27.9
158,500	23.54	24.72	24.74
251,000	22.48	23.4	22.96
398,000	23.98	25.57	24.96
631,000	28.82	32.7	28.54

Table 14 contains the calculated power gain for conjugate-image termination and no internal feedback, that is $h_{12e} = 0$. These calculations were made using equation 31.

Table 14. Calculated power gain in decibels for a neutralized amplifier ($h_{12e} = 0$)

Frequency cycles per second	Transistor 1	Transistor 2	Transistor 3	Transistor 4
1,000	35.83	39.83	37.71	39.0
1,585	35.7	39.72	37.8	39.05
2,510	35.7	39.46	37.8	39.15
3,980	35.47	39.32	37.46	39.1
6,310	35.12	38.48	36.77	38.0
10,000	34.58	37.7	35.75	36.4
15,850	33.45	35.96	34.1	34.8
25,100	31.74	33.75	32.13	32.47
39,800	29.12	31.44	29.86	30.17
63,100	26.86	29.41	29.32	27.93
100,000	24.34	27.08	26.52	25.89
158,500	22.46	24.93	23.52	23.35
251,000	21.62	23.01	22.42	21.89
398,000	22.9	23.52	24.12	23.52
631,000	26.6	23.81	28.08	26.2